Price Categorization, Limited Memory, and Competition

ff fi
fi
fi
fi
fi ff
fi
ff fi
fi
fi
fi

fi

ff 1

fi
fi

fi ff
fi

ff fi

[1]See Alba, Hutchinson and Lynch (1991) for a survey of the information processing literature that examines the effect of memory on consumer choice.

[2] Thus the model is in the spirit of bounded rationality as defined in Simon (1987) where the decision maker makes a rational choice that takes into account the cognitive limitation of the decision maker.

[3] As in Dow (1991), this process is not to be interpreted as one in which consumers search across the firms. Nevertheless, in this paper, we also consider the case in which the decision process of the limited memory consumers involves the initial decision of whether to compare prices at all, after observing the price at the first firm.

fi
fi
fi fi
ff fi
fi fi
fi fi
ff

fi
fi
fi
fi
ff fi
fi
ff
fi
fi
ff
fi
fi fi
fi
ff
ff
ff
ff ff
ff
fi

[4] Alternatively, γ of these consumers can also be assumed to consider purchasing only from one of the two firms, while remaining γ consider the other firm.

[5] Note that we assume that if the observed price at the second firm is exactly identical to the price that is recalled from memory, the consumer will buy at the second firm.

[6] This decision process can also have other interpretations pertaining to the broader class of problems of communication constraints between agents in organizational settings. For example, it can be seen as reflecting communication constraints when the decision making team consists of a team of two agents. Agent 1 observes the price in the first firm and sends a message to agent 2, who then has the discretion to decide after observing the price at firm 2. Here the constraint can be interpreted as a limit on the set of words/messages that can be sent.

[7] Indeed, the possibility of consumers being able to costlessly note down the price provides a possible justification for the consideration of perfect memory and limited memory consumer segments. The perfect memory consumers can be seen as those consumers who can costlessly and perfectly note down all the relevant "full price" information that is necessary for across firm comparisons, while the limited memory consumers find it costly (or are unable) to note down the relevant information and therefore rely on their recall of the information.

fi

n

n

fi

fi

ffi

fi

m_{ij} E $p_j \mid p_j \in C_i$. $k_{(n+1)} \geq m_{(n+1)j} > k_n \geq m_{nj} > \cdots > k_1 \geq m_{1j} > k_0.$

ﬀ

ﬁ ﬁ ﬁ

ﬁ

ﬁ

ﬁ n

n ,

n ,

ﬁ

n ﬁ

ﬁ ﬁ ﬂ

ﬁ

n ﬀ {s Þ

[8] The reasons are: i) if the price of say Firm 1 is high enough, Firm 2 will have incentive to undercut the price to attract the 2α and/or 2β segments; ii) in the other cases, Firm 2 will have the incentive to increase its price to the reservation price, 1, and sell to just the γ consumers.

fi

$$p \quad \Pi \quad \gamma \quad w\beta$$

$$p \quad v \quad \Pi \quad \gamma \quad w\beta \quad w\alpha \; v$$

$$p \quad m \quad \Pi \quad \gamma \quad w\beta \quad \beta \quad w\alpha \; m$$

$$p \quad b \quad \Pi \quad \gamma \quad w\beta \quad \beta \quad \alpha \; b$$

, fi

m

m fi v fi

fi fi

fi

fi Π

$$W_j \; p \quad W \; p \quad \begin{cases} & p > \\ \frac{\Pi}{2\alpha}\frac{1-p}{p} & \geq p \geq v \\ \frac{\Pi}{2\alpha}\frac{1-v}{v} & v \geq p \geq m \\ \frac{\Pi}{2\alpha}\frac{1-p}{p} - \frac{\beta}{2\alpha} & m \geq p \geq b \\ & b > p \end{cases}$$

m

$$m \quad \int p \frac{d}{dp} \quad - W \; p \; dp \quad \frac{\Pi}{\alpha} \quad \frac{m}{b} \quad \frac{\Pi}{\alpha} \quad \overline{v}$$

$v \quad \frac{\gamma+w\beta}{\gamma+w\beta+2w\alpha}$, $m \quad \frac{\gamma+w\beta}{\gamma+w\beta+\beta+2w\alpha}$ $b \quad \frac{\gamma+w\beta}{\gamma+w\beta+\beta+2\alpha}$.

w,

v

$$\left(\frac{\gamma \quad w\beta \quad w\alpha \quad \gamma \quad w\beta \quad \beta \quad \alpha}{\gamma \quad w\beta \quad \gamma \quad w\beta \quad \beta \quad w\alpha} \right) \quad \frac{\alpha}{\gamma \quad w\beta \quad \beta \quad w\alpha}$$

w $\left[\;, \frac{1}{2}\right]$.

fi fi ff fi

fi fi ff

fi ff ff
ff

ff

$\alpha \rightarrow$
v , m b $\frac{\gamma}{\gamma+\beta}$ γ. p
p γ. W p
$\alpha \rightarrow$ w $\rightarrow$. m γ
fi γ.

n

fi b, b $\frac{\gamma}{1-\gamma}$
fi p_j b fi
p_j γ β α γ, fi p_j
n b, .
k_0 b $\frac{\gamma}{1-\gamma}$. fi

fi $\bigcup_{i=1}^{n+1}$ $b_i, m_i \cup v_i, k_i$ $k_i > v_i > m_i > b_i > k_{i-1}$
b_i k_{i-1}

W p Pr p $\geq$ p fi j , p.
W p W p .
fi m
γ βW k $_{-1}$ α β W m . m ², ² $\rightarrow$ fi
γ βW k α β W m . m ,
m v > m . fi
i b . fi
π b b γ βW v βW b αW b π k $_{-1}$ k $_{-1}$ γ βW v $_{-}$ βW b αW b .

w_i W v_i s_i W b_i i , ⋯ , n , ﬁ s_{n+2}
s_i i ⋯ , n .
ﬁ ﬀ
ﬁ i , ⋯ , n

p_j k_i Π γ $w_i\beta$ $s_{i+1}\beta$ $s_{i+1}\alpha$ k_i
p_j v_i Π γ $w_i\beta$ $s_{i+1}\beta$ $w_i\alpha$ v_i
p_j m_i Π γ $w_i\beta$ $s_i\beta$ $w_i\alpha$ m_i
p_j b_i Π γ $w_i\beta$ $s_i\beta$ $s_i\alpha$ b_i

k_i ﬁ
ﬁ ﬁ
m_i
m_i ﬁ ﬁ
ﬁ b_{i+1} m_{i+1} v_i ﬁ
ﬁ
ﬁ m_i ﬁ
ﬁ
b_i m_i ﬁ ﬁ
m_i
ﬁ ﬁ
m_i
i m_i $\int_{b_i}^{k_i} p \frac{d}{dp}$ $-$ W p dp.
ﬁ k_{n+1} ﬁ π γ $w_{n+1}\beta$ ﬁ
ﬁ

w_{n+1} . α

i p k_i p m_i k_i

$$p_j \quad m_i \quad \pi \quad \gamma \quad s_{i+1}\beta \quad s_i\beta \; m_i \quad \forall i \quad , \cdots , n$$

fi fi [9]

$\alpha \to$

n

fi p_i m_i $\frac{2}{\frac{1}{k_{i-1}} + \frac{1}{k_i}}$ m_i $\frac{\gamma}{2\beta} \; \frac{1}{k_{i-1}} - \frac{1}{k_i}$ i

$, \cdots , n$ $\{k_i\}_{i=1}^{n}$ *ff*

$k_{n+1} > k_n > \cdots > k_1 > k_0 \quad \frac{\gamma}{1-\gamma}$.

α

fi $\alpha \to$

fi m fi $k \; \forall i \quad , \cdots , n$.

p k_{+1} fi γ fi p k fi

fi

$$\pi \quad \gamma \quad s_{+1}\beta \; k \quad \gamma \quad i \quad , \; , \cdots , n$$

[9]In empirical studies it has been observed that the distributions of prices are typically such that most of the probability mass is concentrated around a small number of price points (see for example, Villas-Boas 1995 and Rao et. al. 1995).

$$s \quad \frac{\gamma \quad - k_{\ -1}}{\beta k_{\ -1}} \quad i \quad , \ , \cdots n \quad ,$$

$s_1 \quad \frac{y(1- \ _0)}{2\beta \ _0}$ fi Pr m $s \ - s_{\ +1}$

$$m \quad \frac{\gamma \quad - k_{\ -1}}{\beta k_{\ -1}} - \frac{\gamma \quad - k}{\beta k} \quad \frac{\gamma}{\beta} \ \frac{}{k_{\ -1}} - \frac{}{k}$$

$$m \quad \frac{\gamma}{\gamma \quad s \ \beta \quad s_{\ +1}\beta} \quad \frac{\gamma}{\gamma \quad \frac{y(1- \ _{i-1})}{2 \ _{i-1}} \quad \frac{y(1- \ _i)}{2 \ _i}} \quad \frac{}{\frac{1}{_{i-1}} \quad \frac{1}{_i}}$$

ff

fi

γ. fi

γ.

fi

fi fi

fi

fi

fi

fi fi

fi fi ff

ff fi

ff k_i^*

ff fi fi γ.

fi

fl

$\alpha \rightarrow$, ff

k_i^* $\frac{\gamma}{}$

$\{m^*\}_{=1}^{+1}$

fi

fi

fi

$\{k_i^*\}_{i=1}^{n}$

fi

$i \quad , \cdots , n$ ff

ff

$$k_i^* - k_{i-1}^* \quad \frac{\gamma}{-\gamma}^{\frac{n+1-i}{n+1}} - \frac{\gamma}{-\gamma}^{\frac{n+2-i}{n+1}} \quad \frac{\gamma}{-\gamma}^{\frac{n+1-i}{n+1}} \left(- \frac{\gamma}{-\gamma}^{\frac{1}{n+1}} \right)$$

ff i fi

$\frac{1-\gamma}{\gamma}^{\frac{n+1-i}{n+1}}$ i.

fi

$\frac{\partial k_i^*}{\partial \gamma} >$. ff

fi $- b$.

ff

$\delta >$, $\exists N$ $n > N$

$p \in \frac{\gamma}{1-\gamma}$, $\exists m_j \; n$ $|p - m_j \; n| < \delta$ $|W_n \; p - W \; p| < \delta$, $W_n \; p$

fi p m_j n

n ff

n, | $-k_n$ n |

$-$ $\frac{\gamma}{1-\gamma}$ $\frac{1}{n+1}$ fi

n > N_1 $\frac{\ln(\frac{\gamma}{1-\gamma})}{\ln(1-\delta)}$ $-$ δ.

$p \in C_j$ n m_j n $\in C_j$ n

δ p. fi

C_j n s_j n $\frac{\gamma}{1-2\gamma}\left(\frac{1}{k_{j-1}(n)} - \right)$ k_{j-1} n ff

C_j n p < m_j n W_n p s_j n W_n p s_{j+1} n .

W p $\frac{\gamma}{1-2\gamma}$ $\frac{1}{p}$ $-$ p < m_j n $|W_n$ p $-$ W p | $|s_j$ n $-$

W p | $\frac{\gamma}{1-2\gamma}\left|\frac{1}{k_{j-1}(n)} - \frac{1}{p}\right|$ $\frac{\gamma}{1-2\gamma}\frac{|p-k_{j-1}(n)|}{k_{j-1}(n)p} < \frac{\gamma}{1-2\gamma}\frac{\delta}{p(p-\delta)} < \frac{\gamma}{1-2\gamma}\frac{\delta}{\frac{\gamma}{1-\gamma}(\frac{\gamma}{1-\gamma}-\delta)}$. p > m_j n

$|W_n$ p $-$ W p | $|s_{j+1}$ n $-$ W p | $\frac{\gamma}{1-2\gamma}\left|\frac{1}{k_j(n)} - \frac{1}{p}\right| < \frac{\gamma}{1-2\gamma}\frac{\delta}{p\times p} < \frac{\gamma}{1-2\gamma}\frac{\delta}{p(p-\delta)}$. fi η

$\frac{\gamma}{1-2\gamma}\frac{\eta}{\frac{\gamma}{1-\gamma}(\frac{\gamma}{1-\gamma}-\eta)} < \delta$ n > N_2 $\frac{\ln(\frac{\gamma}{1-\gamma})}{\ln(1-\eta)}$ $-$ $|W_n$ p $-$ W p | < δ.

N N_1, N_2 .

fi

fi ΔE_n p $\frac{E(p)-E_n(p)}{E(p)}$

E p

$$\Delta E_n\ p \quad - \frac{n}{\frac{1-\gamma}{\gamma}} \frac{\frac{1-\gamma}{\gamma}\ \frac{1}{n+1}\ -}{\frac{1-\gamma}{\gamma}\ \frac{1}{n+1}}$$

$\frac{\partial \Delta E_n(p)}{\partial n}$ < $\frac{\partial^2 \Delta E_n(p)}{\partial n^2}$ >

n n.

ff fi

$\frac{\partial \Delta E_n(p)}{\partial \gamma}$ < $\frac{\partial^2 \Delta E_n(p)}{\partial \gamma^2}$ > ,

γ ff $_{n\to\infty}$ ΔE_n p

fi

S_β n

$S - S_\gamma - \pi \quad - \gamma \quad \gamma E_n$ p , fi

n λ_n $\frac{S_\beta - S_\beta(n)}{S_\beta}$ $\frac{2\gamma(E(p)-E_n(p))}{1-4\gamma+2\gamma E(p)}$

$\frac{\partial\lambda_n}{\partial n} <$, $\frac{\partial^2\lambda_n}{\partial n^2} >$, $\frac{\partial\lambda_n}{\partial\gamma} >$, $\frac{\partial^2\lambda_n}{\partial\gamma^2} >$.

n,

n.

$\frac{\partial^2\lambda_n}{\partial n^2} >$

n.

fi

$\frac{\partial S_\beta(n)}{\partial\gamma} <$, $\frac{\partial^2 S_\beta(n)}{\partial\gamma^2} >$

fi

fi

fi

fi

fi

fi ff

fi fi

fi

fi

fi fi

fi fi

fi fi

ffi

ﬃ ﬁ ﬁ ﬀ
ﬁ
$\alpha \rightarrow$ ﬀ
ﬁ ﬁ ﬁ
ﬁ m_i
i >
ﬀ ﬁ
i ﬀ
ﬁ
ﬁ ﬁ
ﬀ

$\alpha >$. n
u
ﬁ ﬁ ﬁ
ﬁ ﬁ m
p > u. ﬁ
ﬁ ﬁ m. b, u ∪ d, m ∪ v,
b < u < d < m < v < ﬁ
ﬁ
n $\alpha >$ i
u
ﬁ n
ﬁ m ﬁ ﬁ

i
ﬁ

ﬀ
ﬁ
ﬁ ﬁ
ﬀ

fi

fi

fi fi

fi

fi

fi

ff

fi ff $\{\mathsf{k}_i\}^n$

fi fi ff n fi
fi
ff ff
ff fi
fi
fi
fi ff
fi
fi
fi
fi
ff
fi ff
ff
ff

fi
fi ff
fi
fi
fi
fi fi
fi ff n fi fi
ff
fi
fi
fi fi
fi
fi
ff
fi fi fi
fi
ff fi
ff

fi
ff
ff
ff
ff fi
ff
ff

ﬀ

ﬃ
27

ﬃ

19

119

15

58

ﬁ

14

26

2

117

61

ﬀ
36

7

101

1

70

ﬁ

ﬁ ﬁ ﬁ ﬁ

ﬁ

ﬁ ﬁ ﬁ

The n case

n . u

ﬁ ﬁ m $p > u$.

b, u ∪ d, m ∪ v, $b < u < d < m < v <$ ﬁ W p

Pr $p \geq p$ f p

$$\pi \quad \gamma \quad \beta W \; m \quad p$$
$$\pi \quad \gamma \quad \alpha W \; v \quad \beta W \; m \; v \quad p \quad v$$
$$\pi \quad \gamma \quad \alpha \quad \beta \; W \; m \quad \beta W \; u \; m \quad p \quad m$$
$$\pi \quad \gamma \quad \alpha W \; d \quad \beta W \; m \quad \beta W \; u \; w \quad p \quad d$$
$$\pi \quad \gamma \quad \alpha W \; u \quad \beta \quad \beta W \; u \; u \quad p \quad u$$
$$\pi \quad \gamma \quad \alpha \quad \beta \quad \beta W \; u \; b \quad p \quad b$$
$$\pi \quad \gamma \quad \alpha W \; p \quad \beta W \; m \; p \quad v < p <$$
$$\pi \quad \gamma \quad \alpha W \; p \quad \beta W \; m \quad \beta W \; u \; p \quad d < p < m$$
$$\pi \quad \gamma \quad \alpha W \; p \quad \beta \quad \beta W \; u \; p \quad b < p < u$$

W m W v W d W u W W b

ﬁ W m h W u g

$$W \; p \quad \frac{\gamma \quad \beta h}{ap} - \frac{\gamma \quad \beta h}{a} \quad v \leq p \leq$$
$$W \; p \quad \frac{\gamma \quad \beta h}{ap} - \frac{\gamma \quad \beta h \quad \beta g}{a} \quad d \leq p \leq m$$
$$W \; p \quad \frac{\gamma \quad \beta h}{ap} - \frac{\gamma \quad \beta \quad \beta g}{a} \quad b \leq p \leq u$$

ﬁ v $\frac{\gamma+\beta}{\gamma+\alpha \quad +\beta}$, m $\frac{\gamma+\beta}{\gamma+\alpha \quad +\beta \quad +\beta}$, d $\frac{\gamma+\beta}{\gamma+\alpha \quad +\beta \quad +\beta}$, u $\frac{\gamma+\beta}{\gamma+\alpha \quad +\beta+\beta}$, b $\frac{\gamma+\beta}{\gamma+\alpha+\beta+\beta}$. fi m W .

$$m \quad \frac{\gamma \quad \beta h}{a} \int^{1} \frac{}{p} dp \quad \int^{\hat{}} \frac{}{p} dp$$
$$\frac{\gamma \quad \beta h}{a} \quad \frac{}{v} \frac{m}{d}$$

fi u W .

$$\int^{\hat{}} u - p \ f \ p \ dp \quad \int u - p \ f \ p \ dp$$
$$\Rightarrow \quad u \quad - h \quad \frac{\gamma \quad \beta h}{a} \quad \frac{m}{d} \frac{u}{b}$$

v, m, d, u, b

$$\frac{\gamma \quad \beta h}{\gamma \quad \alpha h \quad \beta h \quad \beta g} \quad \frac{\gamma \quad \beta h}{a} \quad \frac{\gamma \quad \alpha h \quad \beta h}{\gamma \quad \beta h} \frac{\gamma \quad \alpha g \quad \beta h \quad \beta g}{\gamma \quad \alpha h \quad \beta h \quad \beta g}$$
$$\frac{\gamma \quad \beta h}{\gamma \quad \alpha g \quad \beta \quad \beta g} \quad - h \quad \frac{\gamma \quad \beta h}{a} \quad \frac{\gamma \quad \alpha g \quad \beta h \quad \beta g}{\gamma \quad \alpha h \quad \beta h \quad \beta g} \frac{\gamma \quad \alpha \quad \beta \quad \beta g}{\gamma \quad \beta \quad \beta g}$$

h, g fi $g \geq h$. $\alpha \to$

$$\pi \quad \gamma \quad \beta h \quad \pi \quad \gamma \quad \beta h \ v$$
$$\pi \quad \gamma \quad \beta h \quad \beta g \ m \quad \pi \quad \gamma \quad \beta h \quad \beta g \ d$$
$$\pi \quad \gamma \quad \beta \quad \beta g \ b \quad \pi \quad \gamma \quad \beta \quad \beta g \ u$$

b u, m d, v . $\alpha \to$ W p $\frac{\gamma+\beta}{} - \frac{\gamma+\beta}{}$ v $\leq$ p $\leq$ h W v $\frac{\gamma+\beta}{} - \frac{\gamma+\beta}{}$

$$\frac{\gamma \quad \beta h}{\gamma \quad \alpha g \quad \beta \quad \beta g} \quad - h \quad \frac{\gamma \quad \beta h}{a} \quad \frac{\gamma \quad \alpha g \quad \beta h \quad \beta g}{\gamma \quad \alpha h \quad \beta h \quad \beta g} \frac{\gamma \quad \alpha \quad \beta \quad \beta g}{\gamma \quad \alpha g \quad \beta \quad \beta g}$$

g n m d v u. $\alpha \to$, fi u fi m $\alpha \to$

The General Case of n categories

n $\alpha >$

i

i > , u

fi i > ,

u , fi

v $_{-1}$, u k $_{-1} \in$ v $_{-1}$, u

k $_{-1}$ k $_{-1}$ ε. k $_{-1}$ fi k $_{-1}$

fi k $_{-1}$ ε $\varepsilon \rightarrow$. k $_{-1}$

fi k $_{-1} - \varepsilon$

i > .

α.

i b$_1$, u $\cup$ d, m $\cup$ v$_1$, k$_1$ b$_1$ < u < d < m <

v$_1$ < k$_1$ m u

fi fi

n

ﬀ

$k^* \quad \left(\frac{1}{2}\right)^{+1-} \quad \varepsilon,$ i , ..., n, $\varepsilon \to$ $\varepsilon << \left(\frac{1}{2}\right) \ \forall n,$ ﬁ

$p^* \quad k_1^* \quad \frac{1}{2} \quad \varepsilon.$ ﬁ ﬁ $\pi^* \quad \frac{1}{2} \quad {}^{+1} \quad \frac{\varepsilon}{2},$ n.

ﬁ n

ﬁ

ﬁ p_1 p_2 k_1 ﬁ

ﬀ

ﬁ ﬀ ﬁ ﬁ

$$\Pi_1 \ p_1 \quad k \ , p_2 \quad k \quad \begin{cases} & r > t \\ \frac{r}{2} & r \quad t \\ k & r < t \end{cases} \quad \text{for} \ r, t \quad , ..., n$$

ﬀ

ﬀ

p_1 k_{+1} , p_2 k_{+1} ﬀ $\frac{n+1}{2} < k$

p_1 k , p_2 k $\frac{n}{2} < k_{-1}.$ $\frac{r+1}{2} < k$

r , ..., n. k r , ..., n

$$k > \left(- \right)^{+1-}$$

$k^* \quad \left(\frac{1}{2}\right)^{+1-} \quad \varepsilon$ ﬁ $\varepsilon >$ ﬁ

ﬁ $\pi^* \quad \frac{1}{2} \left(\frac{1}{2}\right) \quad \frac{\varepsilon}{2}$

ﬁ ﬀ

ﬁ γ

ﬁ ﬁ

γ ﬁ

γ

ﬁ $- \ \gamma$ n

$\frac{1}{2(1-\gamma)} > \gamma$

ﬀ $k^* \quad \left(\frac{1}{2(1-\gamma)}\right)^{+1-} \quad \varepsilon,$ i , ..., n, $\varepsilon \to$ $\varepsilon << \left(\frac{1}{2(1-\gamma)}\right) \ \forall n,$

ﬁ $p^* \quad k_1^* \quad \frac{1}{2(1-\gamma)} \quad \varepsilon.$

fi ff p_1 p_2 k_1 fi k_1/
ff fi

$$\Pi_1\ p_1\ \ k\ , p_2\ \ k \quad \begin{cases} \gamma k & r > t \\ \frac{r}{2} & r\ \ t \\ -\gamma\ k & r < t \end{cases} \quad \text{for}\ r, t\ \ ,\ldots,n$$

fi fi
r $,\ldots,n$ p_1 k $, p_2$ k
fi fi
p_1 fi π_1 γ fi

$$\Pi\ \ p_1\ \ k\ , p_2\ \ k\ \ \frac{k}{} > \gamma$$

fi

$$\Pi\ \ p_1\ \ k\ , p_2\ \ k\ \ \frac{k}{} > \ -\gamma\ \ k_{\ -1}$$

$$\Pi\ \ p_1\ \ k_{\ +1}, p_2\ \ k_{\ +1}\ \ - > \ -\gamma\ \ k$$

$$\Pi\ \ p_1\ \ k_1, p_2\ \ k_1\ \ \frac{k_1}{} > \gamma$$

$\frac{1}{2} \geq \gamma$ r $,\ldots,n$ fi fi
fi k_1.
fi r $,..,n$

$$\frac{k}{} \leq \ -\gamma\ \ k_{\ -1}$$

$\frac{1}{2}$ $\frac{n+1}{2} \leq \ -\gamma\ \ k$ $k\ \geq \frac{1}{2(1-\gamma)}$.
$k\ \geq \left(\frac{1}{2(1-\gamma)}\right)^{\ +\ -1}$ fi $k_1 \geq \left(\frac{1}{2(1-\gamma)}\right)$. $\frac{1}{2} \geq \gamma$
k_1 fi

$$k_1 \geq \left(\frac{}{\ -\gamma}\right) \ \geq\ \ \gamma$$

$\gamma < \frac{1}{2}$, $\gamma \geq \frac{\gamma}{1-\gamma}$ $k_1 > k_0$ $\frac{\gamma}{1-\gamma}$.

n q_{+1} j n, n − , n − .., Π γ βq_{+1} γ

$$q_{+1}$$
$$q \quad \frac{\Pi}{\beta}\left(\frac{}{k} - \frac{}{k_{+1}}\right) \quad \frac{\gamma}{\beta}\left(\frac{}{k} - \right)$$
$$q_{-1}$$
$$q_{-2} \quad \frac{\gamma}{\beta}\left(\frac{}{k_{-2}} - \frac{}{k_{-1}}\right)$$
...

$$q_{-1} \quad q \quad \frac{\gamma}{\beta}\left(\frac{}{k_{-1}} - \frac{}{k}\right) \quad \frac{\gamma}{\beta}\left(\frac{}{k} - \right)$$
$$\rightarrow \frac{}{k} \qquad \frac{}{k_{-1}}$$

$$q_{-3} \quad q_{-2} \quad \frac{\gamma}{\beta}\left(\frac{}{k_{-3}} - \frac{}{k_{-2}}\right) \quad \frac{\gamma}{\beta}\left(\frac{}{k_{-2}} - \frac{}{k_{-1}}\right)$$
$$\rightarrow \frac{}{k_{-2}} \qquad \frac{}{k_{-1}} \qquad \frac{}{k_{-3}}$$
...

fi k_0 $\frac{\gamma}{1-\gamma}$

$$k^{*} \quad \frac{\gamma}{-\gamma}^{\frac{n\ +1-i}{n\ +1}} \quad i \quad n \quad , n - \ , n - \ ...$$
$$\frac{}{k^{*}_{-2}} \quad \frac{}{k^{*}_{-1}} \quad \frac{}{k^{*}_{-3}} \quad i \quad n \quad , n, n - \ ...$$

n'

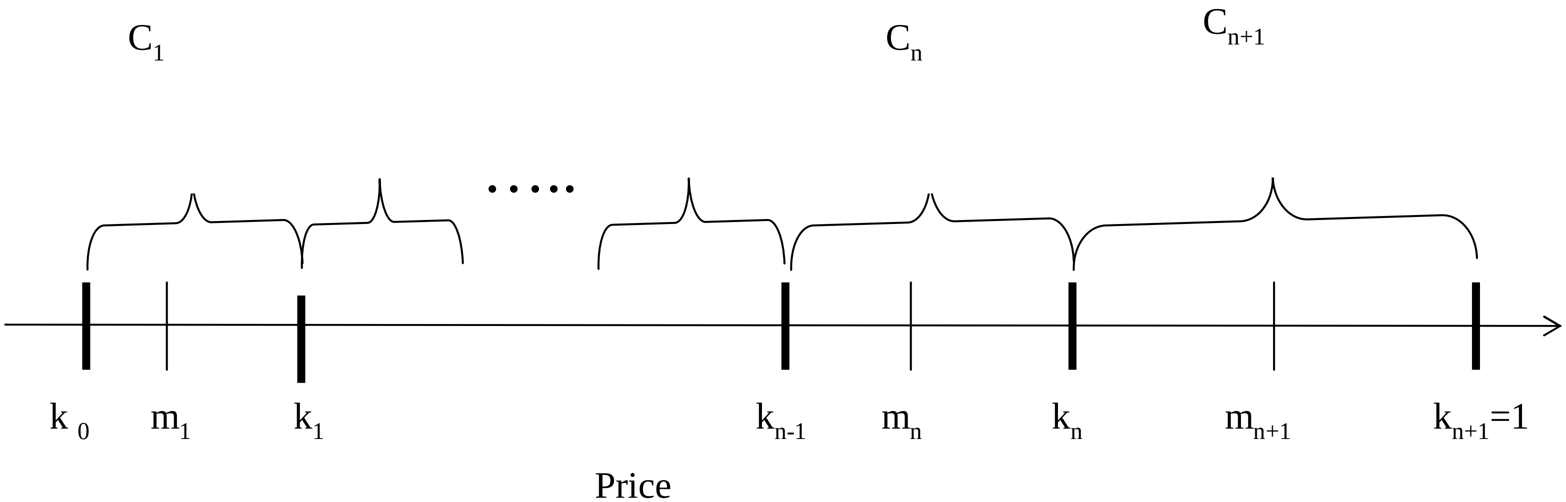

Figure 1: The Categorization Scheme