Adverse Selection and Self-fulfilling Business Cycles*

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Abstract

We introduce a simple adverse selection problem arising in credit markets into a s-tandard textbook real business cycle model. There is a continuum of households and a continuum of anonymous producers who use intermediate goods to produce the final goods. These producers do not have the resources to make up-front payments to purchase inputs and have to finance their working capital by borrowing from competitive financial intermediates. Lending to these producers, however, is risky: honest borrowers will always pay their debt back, but dishonest borrowers will always default. This gives rise to an adverse selection problem for financial intermediaries. In a continuous-time real business cycle setting we show that such adverse selection generates multiple steady states and both local and global indeterminacy, and can give rise to boom and bust cycles driven by sunspots under calibrated parameterization. Introducing reputational effects eliminates defaults and results in a unique but still indeterminate steady state. Finally we generalize the model to firms with heterogeneous and stochastic productivity, and show that indeterminacies and sunspots persist.

Keywords: Adverse Selection, Local Indeterminacy, Global Dynamics, Sunspots.

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1 Introduction

The seminal work of Wilson (1980) shows that in a static model, adverse selection can generate multiple equilibria because of asymmetric information about product quality. The aim of this paper is to analyze how adverse selection in credit markets can give rise to lending externalities that generate multiple steady states and a continuum of equilibria in an otherwise standard dynamic general equilibrium model of business cycles.

To make this point, we introduce a simple type of adverse selection arising in credit markets into a standard textbook real business cycle model. The model features a continuum of households and a continuum of anonymous producers. These producers use intermediate goods to produce the final goods. They do not have the resources to make the up-front payments to purchase intermediate inputs. Therefore, to finance their working capital, they must borrow from competitive financial intermediaries. Lending to these producers however is risky, as some borrowers may default. We assume that there are two types of borrowers (producers). In our baseline model, the honest borrowers will always pay back their loans, while the dishonest borrowers will always default. The financial intermediaries do not know which borrower is honest and which is not. This gives rise to adverse selection: for any given interest rate, the dishonest borrowers have a stronger incentive to borrow. In such an environment, an increase in lending from some optimistic financial intermediaries encourages more honest producers to borrow. The increased quality of borrowers reduces the default risk, which in turn stimulates our model economy. The additional insight from the global dynamics analysis is that even in the absence of local indeterminacy we may still have global indeterminacy, with boom and bust cycles in output under rational expectations. In the model with reputational considerations, we show that the steady state equilibrium is unique, and no default occurs in equilibrium. Nevertheless, perhaps surprisingly, indeterminacy in the form of a continuum of equilibria persists.

Adverse selection in the credit market seems to be a realistic feature, both in poor and rich countries.[1] Our model has several implications that are supported by empirical evidence. First, a large literature has documented that credit risk is countercyclical and has far-reaching macroeconomic consequences. For instance, Gilchrist and Zakrajšek (2012) find that a shock to credit risk leads to significant declines in consumption, investment, and output. Pintus, Wen and Xing (2015) show that interest rates faced by US firms move countercyclically and lead the business cycle. These facts are consistent with our model's predictions. Second, our model delivers a countercyclical markup, an important empirical regularity well documented in the literature. Because of information asymmetry, dishonest borrowers enjoy an information rent. However, when the average quality of borrowers increases due to higher lending, this information rent is diluted. So the measured markup declines, which is critical to sustaining indeterminacy by bringing about higher real wages, a positive labor supply response, and a higher output that dominates the income effect on leisure. Third, our extended model in Section 4 can explain the well-known procyclical variation in productivity. The procyclicality of average quality in the credit market implies that resources are reallocated towards producers with lower credit risk when aggregate output increases. The improved resource allocation then raises productivity endogenously. The procyclical endogenous TFP immediately implies that increases in inputs will lead to a more than proportional increase in total aggregate output, mimicking aggregate increasing returns. This effective increasing returns to scale arises only at the aggregate level. It is also consistent with the results of Basu and Fernald (1997), who find slightly decreasing returns to scale for typical two-digit industries in the US, but strong increasing returns to scale at the aggregate level.

Related Literature Our paper is closely related to several branches of literature in macroeconomics. First, our paper builds on a large strand of literature on the possibility of indeterminacy in RBC models. Benhabib and Farmer (1994) point out that increasing returns to scale

[1] See Su (2007) for evidence of syndicated loans in the US and Karlan and Zinman (2009) for evidence from eld experiments in South Africa.

can generate indeterminacy in an RBC model. The degree of increasing returns to scale in production required to generate indeterminacy, however, is considered be too large (See Basu and Fernald (1995, 1997)). Subsequent work in the literature has introduced additional features to the Benhabib-Farmer model that reduce the degree of increasing returns required for indeterminacy. In an important contribution, Wen (1998) adds variable capacity utilization and shows that indeterminacy can arise with a magnitude of increasing returns similar to that in the data. Gali (1994) and Jaimovich (2007) explore the possibility of indeterminacy via countercyclical markups due to output composition and firm entry respectively. The literature has also shown that models with indeterminacy can replicate many of the standard business cycle moments as the standard RBC model (see Farmer and Guo (1994)). Furthermore, indeterminacy models may outperform the standard RBC model in many other dimensions. For instance, Benhabib and Wen (2004), Wang and Wen (2008), and Benhabib and Wang (2014) show that models with indeterminacy can explain the hump-shaped output dynamics and the relative volatility of labor and output, which are challenges for the standard RBC models. Our paper complements this strand of literature by adding adverse selection as an additional source of macroeconomic indeterminacy. The adverse selection approach also provides a micro-foundation for increasing returns to scale at the aggregate level. Indeed, once we specify a Pareto distribution for firm productivity, our model in Section 4 is isomorphic to those that have a representative-firm economy with increasing returns. It therefore inherits the ability to reproduce the business cycle features mentioned above without having to rely on increasing returns.[2]

Second, our paper is closely related to a burgeoning literature that study the macroeconomic consequences of adverse selection. Kurlat (2013) builds a dynamic general equilibrium model with adverse selection in the second-hand market for capital assets. Kurlat (2013) shows that the degree of adverse selection varies countercyclically. Since adverse selection reduces the efficiency of resource allocation, a negative shock that lowers aggregate output will exacerbate adverse selection and worsen resource allocation efficiency. So the impact of the initial shocks on aggregate output is propagated through time. Like Kurlat (2013), Bigio (2014) develops an RBC model with adverse selection in the capital market. As firms must sell their existing capital to finance investment and employment, adverse selection distorts both capital and labor markets. Bigio (2014) shows that the adverse selection shock widens a dispersion of capital quality, exacerbates the distortion, and creates a recession with a quantitative pattern similar to

[2] Liu and Wang (2014) provide an alternative mechanism to generate increasing returns via nancial constraints.

that observed during the Great Recession of 2008. Our model generates similar predictions to Kurlat (2013) and Bigio (2014). First, adverse selection is also countercyclical in our model, so the propagation of fundamental shocks via adverse selection, as highlighted by Kurlat (2013) is also present in our model. Second, in our model, adverse selection in the credit markets naturally creates the distortions to both capital and labor inputs. Introducing stochastic and heterogeneous productivities into our extended model in Section 4 aggravates adverse selection, and makes the economy more vulnerable to self-fulfilling expectation-driven fluctuations. While Kurlat (2013) and Bigio (2014) emphasize the role of adverse selection in propagating business cycles shocks, our paper complements their work by showing that adverse selection generates multiple steady states and indeterminacy, and hence can be a source of large business cycle fluctuations driven by self-fulfilling expectations.[3] It is worth noting that, all of the above adverse selection in that model can induce endogenous TFP, amplification, aggregate increasing returns to scale and a continuum of equilibria. Section 5 concludes.

2 The Baseline Model

Time is continuous and proceeds from zero to infinity. There is an infinitely-lived representative household and a continuum of final goods producers. The final goods producers purchase intermediate goods as input to produce the final good, which is then sold to households for consumption and investment. The intermediate goods are produced by capital and labor in a competitive market. We assume no distortion in the production of intermediate goods. Final goods firms do not have resources to make up-front payments to purchase intermediate goods before production takes place and revenues from sales are realized. They must therefore borrow from competitive financial intermediaries (lenders) to finance their working capital. Lending to these final goods producers is risky, as they may default. We assume that there are two types of producers (borrowers): honest borrowers who have the ability to produce and will always pay back the loan after the production, and dishonest borrowers who will fully default on their loan. The lenders do not have information about the borrower types. They make loans to firms by with the adverse selection problem in mind. We begin by assuming that all trade is anonymous so we exclude the possibility of reputation effects. We relax these strong assumptions in Section 3, where we introduce reputation effects.

2.1 Setup

Households The representative household has a lifetime utility function

$$\int_0^\infty e^{- \ t} \left(\log(C_t) - \frac{N_t^{1+}}{1+} \right) dt \tag{1}$$

where > 0 is the subjective discount factor, C_t is the consumption, N_t is the hours worked, > 0 is the utility weight for labor, and ≥ 0 is the inverse Frisch elasticity of labor supply. The household faces the following budget constraint

$$C_t + I_t \leq R_t u_t K_t + W_t N_t + \Pi_t; \tag{2}$$

where R_t, W_t and Π_t denote respectively the rental price, wage and total profits from all the firms and financial intermediaries. As in Wen (1998) we introduce an endogenous capacity utilization rate u_t. As is standard in the literature, the depreciation rate of capital increases with the capacity utilization rate according to

$$(u_t) = {}_0 \frac{u_t^{1+}}{1+}; \qquad (3)$$

where $^0 > 0$ is a constant and > 0.[4] Finally, the law of motion for capital is governew.1 0 Td]TJn591 Tf 38.6

interest rate $R_{ft} > 1$. Hence the profit from borrowing and producing for a honest producer is given by

$$\Pi_t^h = (1 - R_{ft} P_t)\,\Phi. \tag{8}$$

Denote by s_t the measure of honest producers who invest in their projects:

$$s_t = \begin{cases} 1 - & \text{if } R_{ft} < \frac{1}{P_t} \\ \in [0, 1 - \;) & \text{if } R_{ft} = \frac{1}{P_t} \\ 0 & \text{if } R_{ft} > \frac{1}{P_t} \end{cases}. \tag{9}$$

The total demand for intermediate goods is hence given by

$$X_t = s_t \Phi. \tag{10}$$

Since each firm also produces Φ units of the final goods, the total quantity of the final goods produced is:

$$Y_t = s_t \Phi = X_t \tag{11}$$

Intermediate goods The intermediate goods is produced by capital and labor with the technology

$$X_t = A \tilde{K}_t \, N_t^{1-}, \tag{12}$$

where $\tilde{K}_t = u_t K_t$ is total capital supply from the households. In a competitive market the profit of producers is $\Pi_t^x = P_t A \tilde{K}_t \, N_t^{1-} - W_t N_t - R_t \tilde{K}_t$. The first-order conditions are

$$R_t = P_t \frac{X_t}{\tilde{K}_t} = P_t \frac{X_t}{u_t K_t}, \tag{13}$$

$$W_t = P_t (1 - \;) \frac{X_t}{N_t}. \tag{14}$$

Under competition profits are zero, so $\Pi_t^x = 0$, and $W_t N_t + R_t u_t K_t = P_t X_t$.

Financial Intermediaries The financial intermediaries are also operated under competition. Anticipating a fraction Θ_t of the loan will be paid back, the interest rate is then given by

$$R_{ft} = \frac{1}{\Theta_t}. \tag{15}$$

So the financial intermediaries earn zero profit. The honest producers altogether borrow $X_t P_t$ of working capital and the dishonest producers altogether borrow ΦP_t as working capital. Since only the honest producers pay back their loan, the average payback rate is

$$\Theta_t = \frac{X_t P_t}{\Phi P_t + X_t P_t} = \frac{X_t}{\Phi + X_t}. \tag{16}$$

2.2 Equilibrium

We focus on an interior solution so $R_{ft} = \frac{1}{P_t}$.[5] In equilibrium, the total profit is simply $P_t\Phi.$ Hence the total budget constraint becomes

$$C_t + I_t = P_t X_t + \quad P_t \Phi. \tag{17}$$

Since $P_t = \frac{1}{R_{ft}} = \Theta_t = \frac{X_t}{\Phi + X_t}$, the above equation can be further reduced to

$$C_t + I_t = P_t X_t + \quad P_t \Phi = X_t = Y_t. \tag{18}$$

We then obtain the resource constraint,

$$C_t + \dot{K}_t = Y_t - \quad (u_t) K_t. \tag{19}$$

The inverse of markup, using equation (18), is therefore is given by:

$$_t \equiv 1 - \frac{\Pi_t}{Y_t} = 1 - \frac{P_t \Phi}{X_t} = \Theta_t = P_t;$$

as $\; _t = \Theta_t$, it then also represents the average quality of the borrowers in the credit market. Finally, the rental price of capital is given by

$$R_t = \quad _t \cdot \frac{Y_t}{u_t K_t}. \tag{20}$$

Likewise, the wage rate is given by

$$W_t = \quad _t \cdot \frac{(1 - \quad) Y_t}{N_t}. \tag{21}$$

Equations (5), (6) and (7) then become

$$N_t \;=\; \frac{1}{C_t} \quad (1 - \quad) \; _t \frac{Y_t}{N_t}; \tag{22}$$

$$\frac{\dot{C}_t}{C_t} = \quad _t \frac{Y_t}{K_t} - \quad (u_t) - \quad ; \tag{23}$$

$$_t \frac{Y_t}{u_t K_t} = \; {}^0 u_t = (1 + \quad) \frac{(u_t)}{u_t}. \tag{24}$$

Then we have

$$u_t = \left(\frac{_t Y_t}{{}^0 K_t} \right)^{\frac{1}{1+\theta}}; \tag{25}$$

[5] We assume, without loss of generality, that7.9701 T38,/F64 10.9091 Tf 34.546 0 Td [( )]TJ/F8 10.9091 Tf 5.424 0 Td [())]TJ/F

and thus

$$\frac{\dot{C}_t}{C_t} = \frac{\quad}{1+} \qquad {}_t\frac{Y_t}{K_t} - \ : \tag{26}$$

Equation (16) then becomes

$$_t = \frac{Y_t}{\Phi + Y_t} \tag{27}$$

Finally the aggregate production function becomes

$$Y_t = A\left(u_t K_t\right) \quad N_t^{1-} \ : \tag{28}$$

In short, the equilibrium can be characterized by equations (22), (23), (24), (28), (19) and (27). These six equations fully de-27mine the dynamics of the six variables $C_t; K_t; Y_t; u_t; N_t$ and $\ _t$.

Equation (27) implies that $\ _t$ increases with aggregate output. Note that $\frac{1}{\ _t} = \frac{Y_t}{R_t u_t K_t + W_t N_t}$ is the aggregate markup. Therefore the endogenous markup in our model is coun-27cyclical, which is consisten- with the empirical regularity well documen-2d in the li-27ature.[6] The credit spread is given by $R_{ft} - 1 = \ \Phi = Y_t$, moving in a coun-27cyclical fashion as in the data.

The coun-27cyclical markup has importan- implications. For example, i- can make hours and the real wage move in the same direction. To see this, suppose N_t increases, so that output increases. Then according to equation (27), the marginal cost $\ _t$ increases as well, which in turn raises the real wage in equation (21). If the markup is a constan-, then the real wage would be proportional to the marginal product of labor and would fall when hours increase. Note also that when $\ = 0$, *i:e:*, there is no adverse selection in the credit markets, equation (27) implies that $\ _t = 1;$ and our model simply collapses in-o a standard real business cycle model. The markup is $1 = \ _t > 1$ if and only if dishones- firms obtain ren- due -o information asymmetry.

2.3 Steady State

We first study the steady state of the model. We use Z -o denote the steady state of variable Z_t. To solve the steady state, we first express all other variables in -27ms of and then we solve as a fixed-poin- problem. Combining equations (23) and (24) yields

$$^0 u^{\ +1} - \frac{^0 u^{\ +1}}{1+} = \ ;$$

or $u = \ \frac{1}{0} - (1 + \ )^{\frac{1}{1+\theta}}$. Note that u only depends on $\ ^0$, and . Therefore, without loss of generality, we can set $\ ^0 = -$

(1 1+. *Note* ,

depreciation rate then is $(u) = \;\; = $. Given , we have

$$k_y = \frac{K}{Y} = \frac{}{+ \;=} = \frac{}{(1+\;)}; \quad (29)$$

$$c_y = 1 - \; k_y = 1 - \frac{}{1+}; \quad (30)$$

$$N = \left(\frac{(1-\;)}{1-\frac{}{1+}} \frac{1}{} \right)^{\frac{1}{1+\gamma}}; \quad (31)$$

$$Y = A^{\frac{1}{1\;\alpha}} \left(\frac{}{(1+\;)} \right)^{\frac{\alpha}{1\;\alpha}} \left(\frac{(1-\;)}{1-\frac{}{1+}} \frac{1}{} \right)^{\frac{1}{1+\gamma}} \equiv Y(\;). \quad (32)$$

Then we can use equation (27) to pin down from

$$\bar{\Phi} \equiv \;\Phi = \frac{1-}{} \cdot Y(\;) \equiv \Psi(\;); \quad (33)$$

where the left-hand side is the total debt obligation of the dishonest borrowers, and the right hand-side is the maximum amount of bad loans that the credit market can tolerate under adverse selection, given that the average credit quality is . The total loss from these dishonest borrowers equals $\Phi = \;\Phi P R_f$ is exactly compensated from interest gain from the honest borrowers, $\frac{1-}{} \cdot Y(\;) = (R_f - 1)Y(\;)$; if equation (33) holds. When $\frac{}{1-} + \frac{1}{1+} > 1$, $\Psi(\;)$ is a non-monotonic function of since $\Psi(0) = 0$ and $\Psi(1) = 0$. On the one hand, if the average credit quality is 0, the total supply of credit will be zero, and hence no lending will possible. On the other hand, if the average quality is one, *i.e.*, $= 1$, then by definition no bad loan will be made. So given $\bar{\Phi}$, there may exist two steady state values of . Denote $\Psi_{\max} = \max_{0 \leq \; \leq 1} \Psi(\;)$, and $^* = \arg\max_{0 \leq \; \leq 1} \Psi(\;)$. Then we have the following lemma regarding the possibility of multiple steady state equilibria.

Lemma 1 *When $0 < \bar{\Phi} < \Psi_{\max}$, there exists two steady states that solve $\bar{\Phi} = \Psi(\;)$.*

It is well known that adverse selection can generate multiple equilibria in a static model (see, e.g., Wilson (1980)). So it is not surprising that our model has multiple steady state equilibria. A credit expansion by financial intermediaries invites more honest firms to borrow and produce. The increased quality of borrowers reduces the default risk, which then stimulates more lending from other financial intermediaries. In turn, the interest rate charged by financial intermediaries decreases, bringing down the production cost. This triggers an output expansion, and further encourages credit supply from the households, and thus generates more future lending. In a nutshell, lending externality exists both intratemporally and intertemporally. We will show that this type of lending externality generates a new type of multiplicity, which shares some similarities with the indeterminacy literature following Benhabib and Farmer (1994).

2.4 Local Dynamics

A number of studies have explored the role of endogenous markup in generating local indeterminacy and endogenous fluctuations (see e.g., Jaimovich (2006) and Benhabib and Wang (2013)). Following the standard practice, we study the local dynamics around the steady state.

Note that at the steady state and $\bar{\Phi}$ are linked by $\bar{\Phi} = \Psi(\ )$, so we can parameterize the steady state either by $\bar{\Phi}$ or . We will use as it is more convenient for the study of local dynamics. Denote by $\hat{x}_t = \log X_t - \log X$ the percent deviation from its steady state. First, we log-linearize equation (27) to obtain

$$\hat{\ }_t = (1-\ )\hat{y}_t \equiv \ \hat{y}_t; \tag{34}$$

which states that the percent deviation of the marginal cost is proportional to output. Log-linearizing equations (28) and (24) yields

$$\hat{y}_t = \frac{\ \hat{k}_t + (1+\ )(1-\ )\hat{n}_t}{1+\ -(1+\ )\ } \equiv a\hat{k}_t + b\hat{n}_t; \tag{35}$$

where $a \equiv \frac{\ }{1+\ -(1+\ )\ }$ and $b \equiv \frac{(1+\ )(1-\ )}{1+\ -(1+\ )\ }$. We assume that $1+\ -(1+\ )\ > 0$, or equivalently $\ < \frac{1+\ }{\ } - 1$, to make $a > 0$ and $b > 0$. In general these restrictions are easily satisfied (see section 2.5). We can also substitute out $\hat{n}_t$ after log-linearizing equation (22) to express $\hat{y}_t$ as

$$\hat{y}_t = \frac{a(1+\ )}{1+\ -b(1+\ )}\hat{k}_t - \frac{b}{1+\ -b(1+\ )}\hat{c}_t \equiv \ _1\hat{k}_t + \ _2\hat{c}_t. \tag{36}$$

It is worth mentioning that $a+b = \frac{1+\ -\ }{1+\ -(1+\ )\ } = 1$ if $\ = 0$. Recall that $\ = 0$ corresponds to the case without adverse selection. Thus endogenous capacity utilization alone does not generate an increasing returns to scale effect at the aggregate level. However, $a+b = \frac{1+\ -\ }{1+\ -(1+\ )\ } > 1$ if $\ > 0$. That is, through general equilibrium effects, adverse selection combined with endogenous capacity utilization mimics increasing returns to scale, even though production has constant returns to scale. Furthermore, if $\ >\ $, then $b > 1$. The model can then explain the procyclical movements in labor productivity $\hat{y}_t - \hat{n}_t$ without resorting to exogenous TFP shocks.

The effective increasing returns in production can generate locally indeterminate steady states as in Benhabib and Farmer (1994). If increasing capital can increase the marginal product of capital, given a fixed discount rate, the relative price of capital must fall and the relative price of consumption must rise so that the total return including capital gains or losses equals the discount rate. The increase in the relative price of consumption boosts consumption at the expense of investment, so capital drifts back towards the steady state instead of progressively exploding. The steady state then becomes a sink rather than a saddle, and therefore becomes indeterminate. The mechanism responsible for the increase in the marginal product of capital however is the increase in the supply of labor in response to higher wages that offset diminishing returns to capital in production. In standard contexts this is not possible if leisure is a normal good. In our adverse selection context however the countercyclical markups, associated with where

$$J \equiv \left[\begin{array}{cc} \frac{1+}{} \frac{{}_1 - (1+\ )\ {}_1}{[(1+\ )\ {}_1 - 1]} & \frac{1+}{} \frac{(\ {}_2 - 1) + 1 - (1+\ )\ {}_2}{(1+\ )\ {}_2} \end{array} \right] ; \quad (38)$$

and ${}_1 \equiv \frac{a(1+\ )}{1+\ -b(1+\ )}$, ${}_2 \equiv -\frac{b}{1+\ -b(1+\ )}$, and $\ =\ =$ is the steady state depreciation rate. The local dynamics around the steady state is determined by the roots of J: The model economy exhibits local indeterminacy if both roots of J are negative. Note that the sum of the roots equals the trace of J, and the product of the roots equals the determinant of J. Thus the sign of the roots of J can be observed from the sign of its trace and determinant. The following lemma specifies the sign for the trace and determinant condition for local indeterminacy.

Lemma 2 *Denote* ${}_{\min} \equiv \frac{(1+\ )(1+\ )}{(1+\ )(1-\ )+\ (1+\ )} - 1$ *and* ${}_{\max} \equiv 1 - {}^*$, *then Trace*$(J) < 0$ *if and only if* $\ > {}_{\min}$, *and Det*$(J) > 0$ *if and only if* ${}_{\min} < \ < {}_{\max}$.

According to Lemma 2, our baseline model will be indeterminate if and only if ${}_{\min} < \ < {}_{\max}$. In this case, Trace$(J) < 0$ and Det$(J) > 0$ jointly imply that both roots of J are negative. We summarize this result in the following proposition.

Proposition 1 *The model exhibits local indeterminacy around a particular steady state if and only if*

$${}_{\min} < \ < {}_{\max}. \quad (39)$$

Equivalently, indeterminacy emerges if and only if $\ \in (\ {}_{\min}, \ {}_{\max})$, *where* ${}_{\min} \equiv 1 - {}_{\max} = {}^*$, *and* ${}_{\max} \equiv 1 - {}_{\min}$.

To understand the intuition behind Proposition 1, first note that if $\ > {}_{\min}$, we have

$$1+\ -b(1+\ ) < 1+\ - \frac{(1+\ )(1-\ )}{1+\ -(1+\ {}_{\min})}(1+\ {}_{\min}) = 0. \quad (40)$$

Then the equilibrium elasticity of output with respect to consumption ${}_2$ becomes positive, namely, an autonomous change in consumption will lead to an increase in output. Since capital is predetermined, labor must increase by equation (35). To induce an increase in labor, the real wage must increase enough to overcome the income effect, which is only possible if the increase in markup is large enough. In other words, in equation (34) must be large enough.

We have used the mapping between and steady state output to characterize the indeterminacy condition in terms of the model's deep parameter values. Notice that ${}_{\max} = 1 - {}^*$, where ${}^* \equiv \arg\max_{0 \le\ \le 1} \Psi(\ )$. Since $1 - \bar{}_L > 1 - {}^* = {}_{\max}$, the local dynamics around the steady state associated with $\;=\bar{\ }_L$ are determinate according to Proposition 1. Indeterminacy is only possible in the neighborhood of the steady state associated with $\;=\bar{\ }_H$. The following corollary formally characterizes the indeterminacy condition in terms of $\bar{\Phi}$.

Corollary 1 *Denote* $\bar{\Phi} = \;\Phi$.

1. *If* $\bar{\Phi} \in (0; \Psi(\;_{\max}))$, *then both steady states are saddles.*
2. *If* $\bar{\Phi} \in (\Psi(\;_{\max}); \Psi_{\max})$, *then the local dynamics around the steady state* $\;=\bar{\ }_H$ *exhibits indeterminacy while the local dynamics around the steady state* $\;=\bar{\ }_L$ *is a saddle.*

As suggested by Lemma 1, we focus on the nontrivial region in which $\bar{\Phi} < \Psi_{\max}$. When $\Psi(\;_{\max}) < \bar{\Phi} < \Psi_{\max}$, we have $\;_{\min} = \;^* < \bar{\ }_H < \;_{\max}$, and $\bar{\ }_L < \;_{\min}$. As a result, according to Proposition 1, the steady state $\bar{\ }_H$ exhibits indeterminacy. For the steady state $\;=\bar{\ }_L$, by Lemma 2, we can conclude that the determinant of J is negative. So the two roots of J must have opposite signs and this implies a saddle. But if $0 < \bar{\Phi} < \Psi(\;_{\max})$, we have $\bar{\ }_H > \;_{\max}$ and $\bar{\ }_L < \;_{\min}$. In this case, the determinants of J at both steady states are negative. So both steady states are saddles.

We summarize these different scenarios in Figure 1. The inverted U curve illustrates the relationship between $\;$ and $\bar{\Phi}$ specified in equation (33). In Figure 1, $\;$ is on the horizontal axis and $\bar{\Phi}$ is on the vertical axis. For a given $\bar{\Phi}$, the two steady states $\bar{\ }_L$ and $\bar{\ }_H$ can be located from the intersection of the inverted U curve and a horizontal line through point $(0; \bar{\Phi})$. The two vertical lines passing points $(\;_{\min}; 0)$ and $(\;_{\max}; 0)$ divide the diagram into three regions. In the left and right regions, the determinant of the Jacobian matrix J is negative, implying that one of the roots is positive and the other is negative. So if a steady state $\;$ falls into either of these two regions, it is a saddle. In the middle region, $\mathrm{Det}(J) > 0$ and $\mathrm{Trace}(J) < 0$, and thus both roots are negative. So if the steady state $\;$ falls into the middle region it is a sink which supports multiple self-fulfilling expectation-driven equilibria, or indeterminacy in its neighborhood.

Since $\bar{\Phi} = \;\Phi$, we can reinterpret the above corollary in terms of $\;$, the proportion of dishonest firms. For simplicity, assume Φ is large enough such that $\Phi > \Psi_{\max}$. Denote $\;_L \equiv \Psi(\;_{\max})/\Phi$ and $\;_H \equiv \Psi(\;_{\min})/\Phi = \Psi_{\max}/\Phi$, and thus $0 < \;_L < \;_H < 1$. Then we know that (i) if $\;\in (0; \;_L]$, both steady states are saddles, (ii) if $\;\in (\;_L; \;_H)$, the steady state with $\;=\bar{\ }_L$ is a saddle while the steady state with $\;=\bar{\ }_H$ is a sink, and (iii) if $\;\in [\;_H; 1]$, then there exist no non-degenerate steady state equilibria. As indicated in Lemma 1, the third case

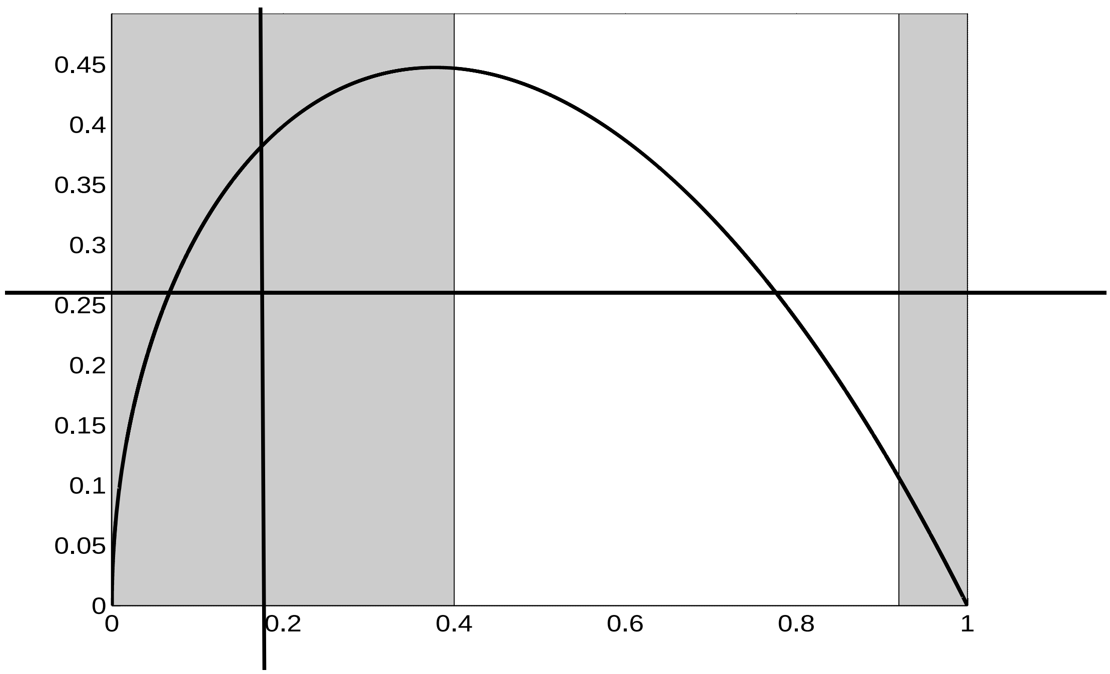

Figure 1: **Multiple Steady States and the Indeterminacy Region**

is the least interesting, and thus we focus on the scenarios in which $< {}_H$. Then the model is indeterminate if the adverse selection problem is severe enough, *i.e.*, $> {}_L$. We summarize the above argument in the following corollary.

Corollary 2 *The likelihood of indeterminacy increases with , the proportion of dishonest firms.*

Arguably, adverse selection is more severe in developing countries. Our study then also suggests that developing countries are more likely to be subject to self-fulfilling expectation-driven fluctuations and hence exhibit higher economic volatility, which is in line with the empirical regularity emphasized by Ramey and Ramey (1995) and Easterly, Islam, and Stiglitz (2000).

2.5 Empirical Possibility of Indeterminacy

We have proved that our model with adverse selection can generate self-fulfilling equilibria in theory. We now examine the empirical plausibility of self-fulfilling equilibria under calibrated parameter values. The frequency is a quarter. We set $= 0.01$, implying an annual risk-free interest rate of 4%. We set $= 0.3$ so the depreciation rate at steady state is 0.033 and the annualized investment-to-capital ratio is 12% (see Cooper and Haltiwanger (2006)). We set $= 0.33$ as in the standard RBC model. We assume that labor supply is elastic, and thus set $= 0$. We normalize the aggregate productivity $A = 1$. We set $= 1.75$ so that $N = \frac{1}{3}$ in the "good" steady state. We set $\overline{\Phi} = \Phi = 0.13$ so that $= \bar{}_H = 0.9$, which is consistent with average profit rate in the data. The associated $\bar{}_L = 0.011$. If we further set $= 0.1$, $i.e.$, the proportion of dishonest borrowers is around 10%, then $\Phi = 1.3$.[8] Consequently, based on our calibration and the indeterminacy condition (39), we conclude that our baseline model does generate self-fulfilling equilibria.

Parameter	Value	Description
	0.01	Discount factor
	0.3	Utilization elasticity of depreciation
	0.033	Depreciation rate
	0.33	Capital income share
	0	Inverse Frisch elasticity of labor supply
	1.75	Coefficient of labor disutility
	0.1	Proportion of firms that produce lemons
Φ	1.3	Maximum firm capacity

Table 1: Calibration

Our calibration uses a delinquency rate of approximately 10%, which of in the same magnitude as in the Great Recession. but is higher than the average delinquency rate in the data (the average is 3.73% from period 1985 to 2013). Delinquency rates do vary over time, however. For example commercial residential mortgages had high delinquency rates during 2009-2013, which spread panic to financial markets through mortgage-backed securities and other derivatives. Nevertheless we will show in Section 3, where we introduce reputation effects, that indeterminacy will arise even if there is no default in equilibrium.

2.6 Global Dynamics

So far we have characterized the steady states and the local dynamics around these steady states. We showed that for some parameters, the equilibrium around one of the steady states is locally determinate. In this section, we analyze the global dynamics and then show that

[8] As shown in equation (33), only the product matters for .

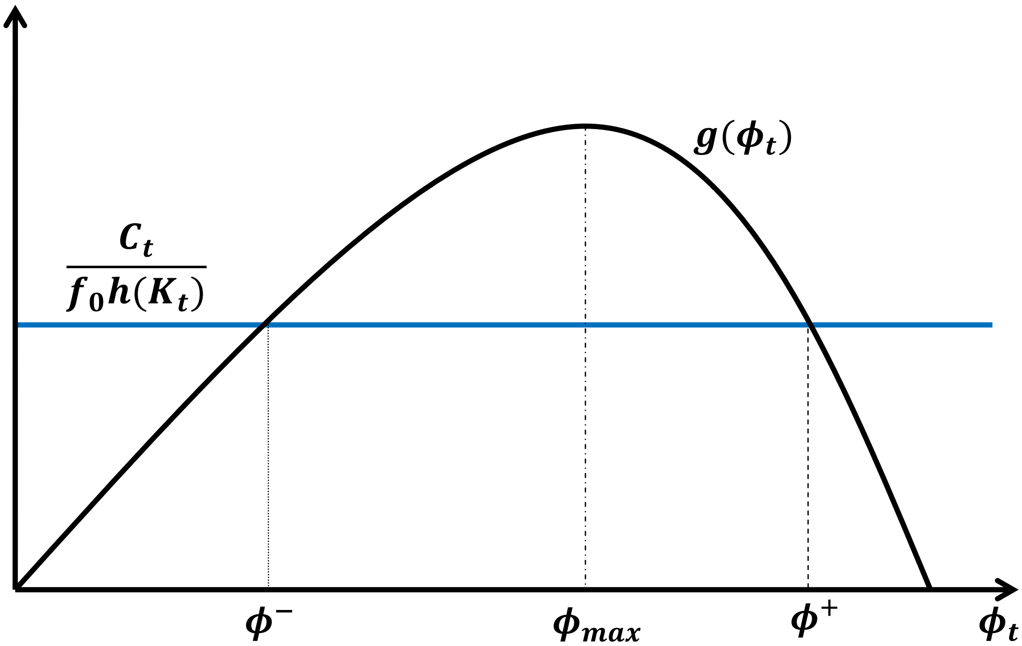

Figure 2: **Illustration of** $\ _t$

global indeterminacy always exists in our model, even in the case where both steady states are saddles and locally determinate.[9]

It is worth noting that it is impossible for us to obtain a two-dimensional autonomous dynamical system that is only related to $(C_t; K_t)$. This is because we do not analytically formulate $\ _t$ in terms of $(C_t; K_t)$. One possible solution is to characterize a three-dimensional dynamical system on $(C_t; K_t; \ _t)$. The main concern, however, is it will be difficult, if not impossible, for us to completely characterize the economic properties of the high-dimensional dynamical system. Fortunately, we can still reduce the dynamical system to a two-dimensional one, but in terms of $(\ _t; K_t)$, as shown in the following proposition.

Proposition 2 *The autonomous dynamical system on* $(\ _t; K_t)$ *is given by*

$$\left(1 \quad + \frac{(1+\ )}{1+}\right)\left(\frac{\ _{\max} \quad _t}{1 \quad _t}\right)\frac{\ _t}{\ _t} + \left(\frac{(1+\ )}{1+}\right)\frac{K_t}{K_t} = (1 \quad)\left(\frac{}{1+} \ _t\frac{Y(\ _t)}{K_t} \quad \right) \tag{41}$$

$$K_t = \left(1 \quad \frac{\ _t}{1+}\right)Y(\ _t) \quad C(\ _t; K_t) \tag{42}$$

[9] For an early growth model with countercyclical markups, multiple steady states and global indeterminacies see Gali (1996).

with $Y_t = Y(\ _t) = \frac{\Phi\ _t}{1-\ _t}$, $_{\max} \equiv 1 - \ _{\min}$, $_{\min}$ *defined in Lemma 2, and*

$$C_t = C(\ _t; K_t) = f_0 \cdot g(\ _t) \cdot h(K_t) \tag{43}$$

where $f_0 = A^{\frac{1+\gamma}{1\ \alpha}}\ _0^{\frac{\alpha(1+\gamma)}{(1+\theta)(1\ \alpha)}}\ \frac{1-}{}$, $h(K_t) = K_t^{\frac{\alpha\theta(1+\gamma)}{(1+\theta)(1\ \alpha)}}$*, and*

$$g(\ _t) = \left[\ _t^{1- \ +\frac{\alpha(1+\gamma)}{1+\theta}} Y(\ _t)^{1- \ -\left(1-\frac{\alpha}{1+\theta}\right)(1+\ )}\right]^{\frac{1}{1\ \alpha}}: \tag{44}$$

As shown in equation (43), we can formulate C_t as a function function of $_t$ and K_t. In turn, We have the following corollary regarding the relationship between equilibrium $_t$ and C_t.

Corollary 3 *For any* $K_t > 0$ *and* $C_t < f_0 \cdot h(K_t) \cdot g(\ _{\max})$*, there exist two possible* $_t$ *values, denoted by* $_t = \ ^+\left(\frac{C_t}{f_0 h(K_t)}\right) > \ _{\max}$ *and* $_t = \ ^-\left(\frac{C_t}{f_0 h(K_t)}\right) < \ _{\max}$*, that yield the same level of consumption defined by (43).*

We illustrate these two possible equilibria $_t$ in Figure 2. The function $g(\ _t)$ has an inverted U shape. It attains the maximum at $_{\max}$. Notice that $g(0) < C_t = [f_0 \cdot h(K_t)] < g(\ _{\max})$, and then by the intermediate value theorem, there exist an $_t^-$ such that $0 < \ _t^- < \ _{\max}$ and $g(\ _t^-) = C_t=[f_0 \cdot h(K_t)]$. Since $g'(\ ) > 0$ for $0 < \ < \ _{\max}$, $_t^-$ must be unique. Similarly, $g(1) < C_t=[f_0 \cdot h(K_t)] < g(\ _{\max})$ and $g'(\ ) < 0$ for $_{\max} < \ < 1$; so there exists a unique $_t^+$ such that $_{\max} < \ _t^+ < 1$ and $g(\ _t^+) = C_t=[f_0 \cdot h(K_t)]$.

As discussed by Lemma 1, the dynamical system on $(\ _t; K_t)$ have two steady states. Motivated by Corollary 1, we consider two cases. In the first case, one of the steady states is a sink and the other is a saddle. In the second case, both steady states are saddles.

2.6.1 Global Dynamics with Local Indeterminacy

We first consider the case in which one steady state is a sink. As illustrated in Figure 1, (the proportion of dishonest firms) is high and both steady state values are smaller than $_{\max}$ in this case. As noted before, there is local indeterminacy around the upper steady state but local determinacy around the lower steady state. However, globally the local steady state is also indeterminate as Figure 3 shows.

In Figure 3, the thick red line is the $\dot{K}_t = 0$ locus and the thick blue line is the $\dot{\ }_t = 0$ locus. The small circles indicate the initial conditions of trajectories. These two loci intersect twice at upper and lower steady states, respectively. For a given K_t, there is a unique level of $_t$ such

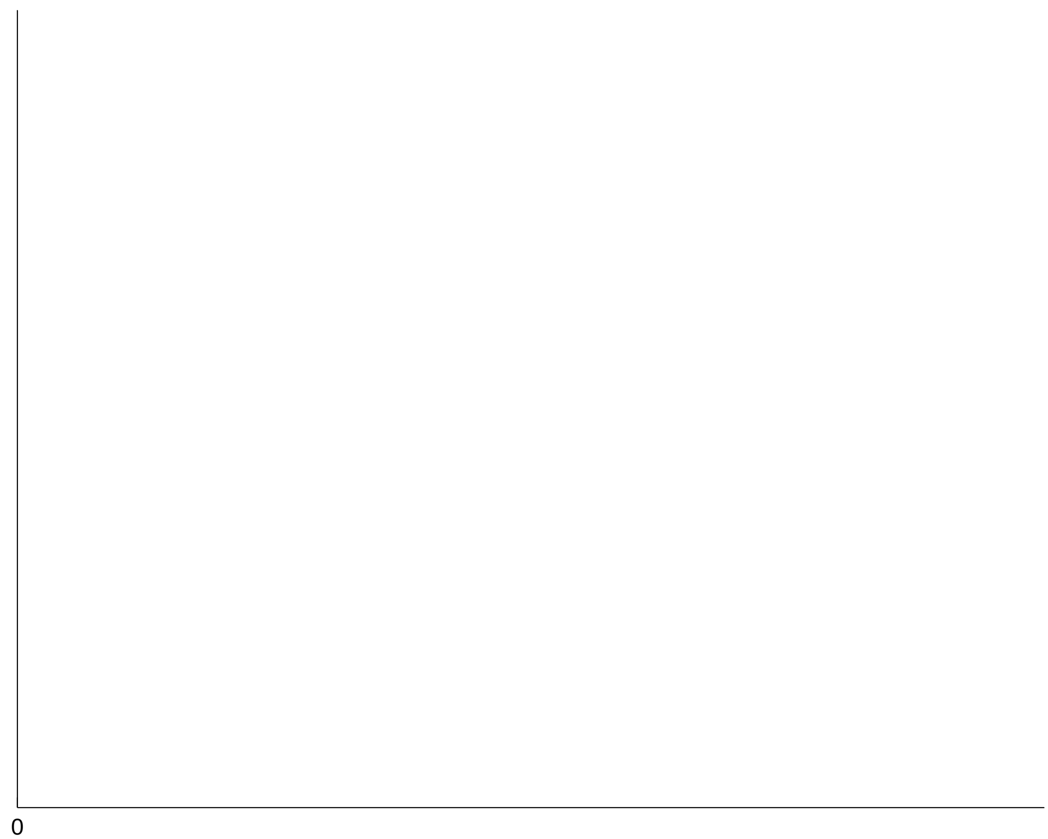

Figure 3: Global Dynamics with One Saddle: A High (We set $= 0:2923$. All the other parameter values are from Table 1.)

that the economy converges to the lower steady state. The function giving the unique level of $_t$ as K_t and converging to the lower steady state is the saddle path in Figure 3, a dashed blue line. If the initial $_t$ is below this saddle path, the economy will eventually converge to the horizontal axis with $_t = 0$ and some positive capital.[10] By equation (43), this implies zero consumption and the transversality condition for households will be violated, so paths starting below this saddle path are ruled out. However, for a given K_t

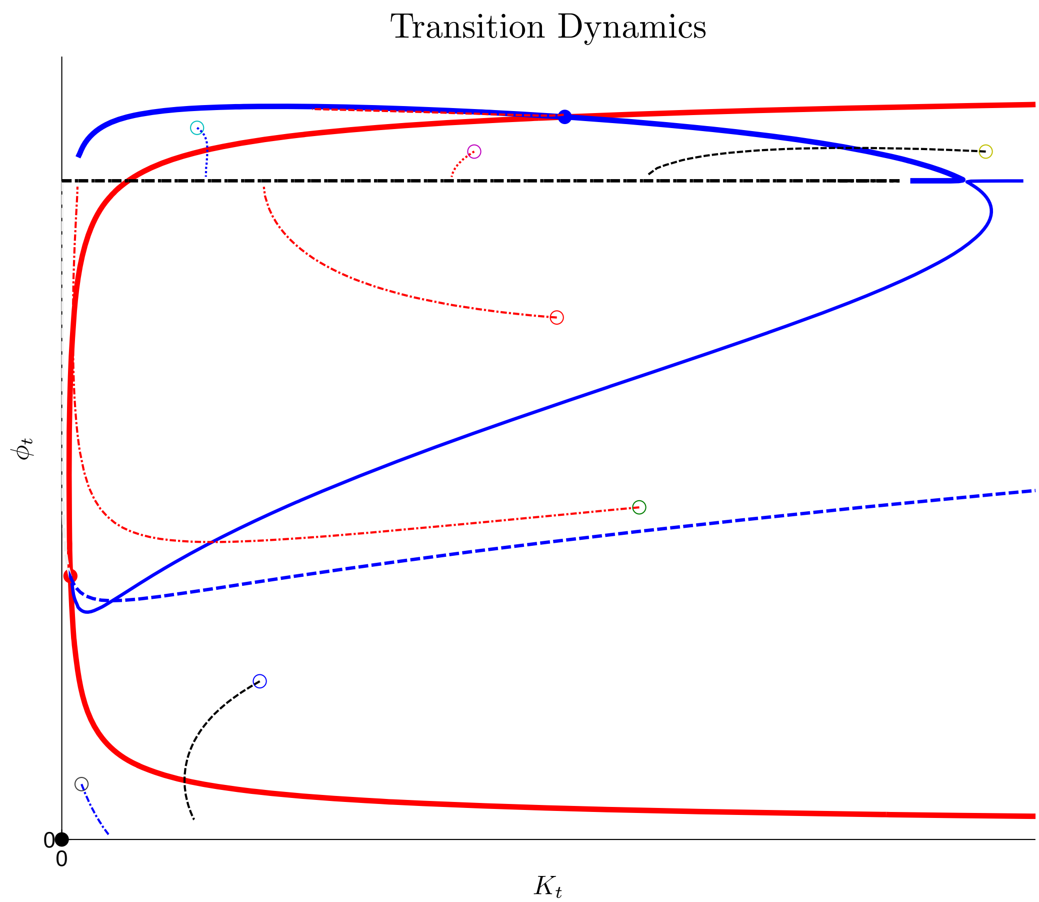

Figure 4: Global Dynamics with One Saddle: Relatively High (We set $= 0{:}62$ and $\Phi = 22$. All the other parameter values are from Table 1.)

2.6.2 Global Dynamics with Two Saddles

In this section we study the global dynamics in the case where both is low such that steady states are saddles, where $\bar{}_H > {}_{\max}$ and $\bar{}_L < {}_{\max}$. We set $= 0.0615$ for the following numerical analysis, including in Figures 5 and 6. All the other parameter values are from Table 1.[13] Figure 4 graphs the two saddle paths associated with these two steady states. This then implies that both steady states are globally indeterminate: for any given K_t, the economy can be on either saddle path. So globally there is still indeterminacy even around each of the steady states. Furthermore, we can create very complicated equilibrium paths if we allow ${}_t$ to jump. We can construct two types of jumps to illustrate the point. The first type of jump in ${}_t$ are deterministic and fully anticipated. Utility maximization then requires consumption to change continuously. That is consumption does not jump when ${}_t$ jumps. Notice that ${}_t = {}_t^+$ and ${}_t = {}_t^-$ yield the same consumption level for a given capital K_t. The economy can always jump from ${}_t = {}_t^+ > {}_{\max}$ to ${}_t = {}_t^- < {}_{\max}$ and back without changing the value of consumption, on a deterministic cycle.

Figure 5 graphs one such possible equilibrium path for each of consumption, investment, output and interest spread once we allow ${}_t$ to jump. The economy starts from point $K = 6.2783$ and $= 0.9717 > {}_{\max}$ and so $C = 0.8723$. With $K = 6.2783$, there exists another $= 0.8249 < {}_{\max}$ that yields $C = 0.8723$. The economy then follows the trajectory according to equations (41) and (42). It takes around 4.41 years for the model economy to reach $K = 11.1719$, $= 0.9270$ and $C = 0.9307$. We then let jump down to a level that allows consumption to remain at 0.9307 upon the jump. By construction, this leads to $= 0.8241 < {}_{\max}$ after the jump. We then let the economy follow the trajectory dictated by equations (41) and (42) again by another 8.02 years to reach $K = 6.2783$, $= 0.8249$ and hence $C = 0.8723$. Notice that the consumption level has returned to its initial level. We then let jump from $= 0.8249$ to $= 0.9717$. Again by construction, consumption does not change immediately. We repeat this process and obtain the deterministic cycles in consumption, investment, output and credit spread in Figure 5. The adverse selection problem is modest when ${}_t > {}_{\max}$, but it becomes much worse when ${}_t < {}_{\max}$. So when ${}_t$ jumps down, there is a collapse in output. Households can insure their consumption by disinvesting capital after ${}_t$ jumps down. In yeneral, there are infinite ways to construct these deterministic cycles, as

[13] To better illustrate the global dynamics with two saddles in Figure 4, we set from 0.33 to 0.62, and from 1.3 to 22. All the other parameter values are from Table 1. The numerical analysis in this section, however, uses standard parameterization in Table 1, only changing the value of from 0.1 to 0.0615.

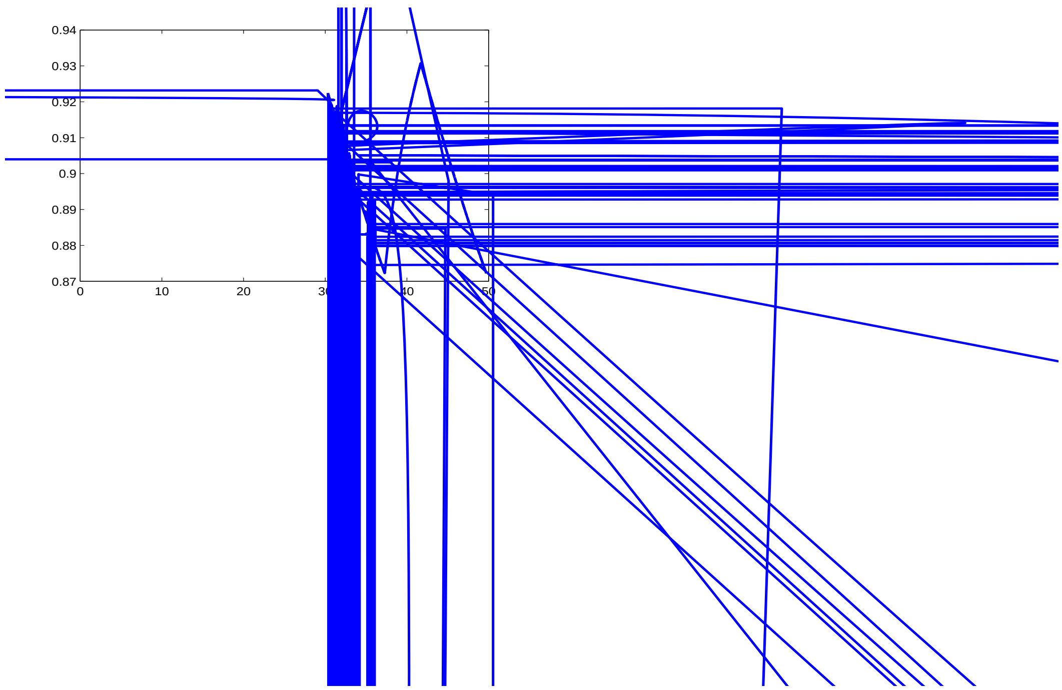

Figure 5: **Deterministic Cycles**

pointed out by Christiano and Harrison (1999).[14] Around the upper steady state, equilibrium $_t$ can take many (possibly infinite values). So the equilibrium around the upper steady-state is still indeterminate, although it is a saddle.

Sunspot Equilibria Finally we can also construct a stochastic sunspot equilibrium by allowing $_t$ to jump randomly. More specifically, we introduce sunspot variables z_t, which take two values, 1 and 0. We assume that in a short time interval dt, there is probability dt that the sunspot variable will change from 1 to 0 and probability $!dt$ that will change from 0 to 1. We construct the equilibrium $_t$ as a function of K_t and sunspot z_t, $i.e.$, $_t = (K_t; z_t)$, such that $(K_t; 1) > (K_t; 0)$. So the equilibrium $_t$ will jump with an anticipated probability when z_t changes its value. When $z_t = 1$, economic confidence is high so adverse selection is modest. But when $z_t = 0$, economic confidence is low, and adverse selection becomes severe. We use the change of z_t from 1 to 0 to trigger an economic crisis, and from 0 to 1 to stop the crisis as

[14] These two $_t$ that yield the same level of consumption correspond to two different branches in the differential equations defined by C_t and K_t. As pointed out by Christiano and Harrison (1999) a model with two branches can display rich global dynamics, regardless of the local determinacy. For example, we can construct an equilibrium with regime switches along these branches. The jumps for $_t$ in the differential equations defined by $_t$ and K_t correspond to the switch of branches in the dynamics defined for C_t and K_t.

economic confidence is restored. We set $\ = 0{:}01$ and $! = 0{:}025$ as an example, which means that the economy will remain in the normal, non-crisis mode with probability $0{:}7143$. Since jumps in $\ _t$ are now stochastic, consumption is exposed to a jump risk. Therefore equation (41) must be modified to take this risk into account. Denote $\ _{1t} = \ (K_t; 1)$ and $\ _{0t} = \ (K_t; 0)$. We then have

$$1 - \quad + \frac{(1+ \quad)}{1+} \quad \frac{\max - \quad {}_{1t}}{1 - \quad {}_{1t}} \quad \frac{\dot{}_{1t}}{{}_{1t}} + \frac{(1+ \quad)}{1+} \quad \frac{\dot{K}_t}{K_t}$$

$$= \ (1 - \quad) \quad \frac{}{1+} \quad {}_{1t}\frac{Y_{1t}}{K_t} - \quad + \quad \frac{g(\ _{1t})}{g(\ _{0t})} - 1 \quad ;$$

for normal non-crisis times. Here the last term $\frac{g(\ _{1t})}{g(\ _{0t})} - 1$ reflects the percentage change in consumption due to the jump from $\ _{1t}$ to $\ _{0t}$ and $Y_{1t} = \ \Phi \ _{1t} = (1 - \ _{1t})$ is aggregate output when $\ _t = \ _{1t}$. Similarly we have

$$1 - \quad + \frac{(1+ \quad)}{1+} \quad \frac{\max - \quad {}_{0t}}{1 - \quad {}_{0t}} \quad \frac{\dot{}_{0t}}{{}_{0t}} + \frac{(1+ \quad)}{1+} \quad \frac{\dot{K}_t}{K_t}$$

$$= \ (1 - \quad) \quad \frac{}{1+} \quad {}_{0t}\frac{Y_{0t}}{K_t} - \quad + ! \quad \frac{g(\ _{0t})}{g(\ _{1t})} - 1 \quad ;$$

in crisis times when $z_t = 0{:}$

It is evident that if $\ = ! = 0$, then $\ _{1t} = \ (K_t; 1)$ and $\ _{0t} = \ (K_t;$

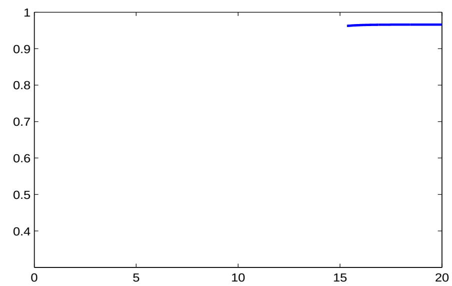

Figure 6: Stochastic Switches between Branches

ante, consumption drops immediately (the top-left panel). Investment (the top-right panel) falls for two reasons: one is to partially offset the fall in output to finance consumption, and the other is due to the decline in the effective return as a result of severe adverse selection in the credit market. The economy stays in crisis mode for about a year and then confidence is restored and the recession is over. Interestingly output and investment both over-shoot when the recession is over, and the longer the economy stays in recession, the larger this overshooting. The longer recession, the smaller is the amount of capital left. So the return to investment is very high, and the households opt to work hard and invest more to enjoy this high return from investment. Figure 6 shows several large boom and bust cycles due to stochastic jumps in the sunspot variables. This shows that there are rich multiple-equilibria in our benchmark model regardless of the model parameters.

3 Reputation

We now study the sensitivity of our indeterminacy results to reputation effects under adverse selection. If firms are not anonymous in the market, they may refrain from defaulting and instead may want to build their reputation. Lenders may also refrain from lending to firms with a bad credit history. Arguably, these market forces can alleviate the asymmetric information problem. So we examine whether the indeterminacy results obtained in our baseline model survives if such reputational effects are taken into account.

We follow Kehoe and Levine (1993) closely in modeling reputation. Firms are infinitely-lived, and can choose to default at any time. Firms that default may, with some probability, acquire a bad reputation and are excluded from the credit market forever. In equilibrium, the fear of loosing all future profits from production discourages firms from defaulting. We will show that self-fulfilling equilibria still exist even if there are no defaults in equilibrium.

To keep the model analytically tractable, we assume that all firms are owned by a representative entrepreneur. The entrepreneur's utility function is given by

$$U(C_{et}) = \int_0^\infty e^{- \ _e t} \log(C_{et}) dt; \tag{45}$$

where C_{et} is the entrepreneur's consumption and $_e$ her discount factor. For tractability, we assume $_e <<$ so that the entrepreneur does not accumulate capital. The entrepreneur's consumption equals the firm's profits,

$$C_{et} = \int_0^1 \Pi_t(i) di \equiv \Pi_t; \tag{46}$$

where $\Pi_t(i)$ denotes the profit of firm i.

Since the only cost of default is the loss of future production opportunities, the price must exceed the marginal cost (also the average cost) of production to be profitable. If the price exceeds the marginal cost, each firm will then have an incentive to produce an infinite amount. To overcome this problem, we assume that the production projects of firms are indivisible, as in the benchmark model, and that they produce to meet the orders they receive. A production project produces a flow of final goods Φ from intermediate goods. Each unit of the final good requires one unit of the intermediate good for its production. The project will be carried out only if the firms receive a purchase order. Denote the total demand for the final good by Y_t. Then a fraction $_t = Y_t / \Phi$ of firms will receive a purchase order. Again we assume that firms need to borrow to finance their working capital. Denote the intermediate good price by P_t, so they need to borrow $P_t \Phi$ to produce Φ.

To illustrate the reputation problem, let us consider a short time interval from t to $t + dt$. We use V_{1t} (V_{0t}) to denote the value of a firm that receives an order (no orders). We can then formulate V_{1t} recursively as

$$V_{1t} = (1 - \ _t)\Phi dt + e^{- \ _e dt} \frac{C_{e;t}}{C_{e;t+dt}} (\ _{t+dt} V_{1t+dt} + (1 - \ _{t+dt}) V_{0t+dt}); \quad (47)$$

where $_t = P_t$ is the unit production cost. If $_t < 1$, then the firm receives a positive profit from production. The second term on the right-hand side is the continuation value of the firms. Since firms areowned by theentrepreneur, the futurevalueis discounted by themarginal utility of the entrepreneur. Since there is no default in equilibrium, the gross interest rate for a working capital loan is $R_{ft} = 1.$

The firms can also choose to default on their working capital, and obtain instantaneous gain of $\Phi \ _t$. However, default comes with the risk of acquiring a bad reputation. Upon default, a firm acquires a bad reputation in the short time interval between t and $t + dt$ with probability dt. In that case, the firm will be excluded from production forever. The payoff for defaulting is hence

$$V_t^d = \Phi dt + e^{- \ _e dt}(1 - \ dt) E_t \frac{C_{e;t}}{C_{e;t+dt}} (\ _{t+dt} V_{1t+dt} + (1 - \ _{t+dt}) V_{0t+dt}). \quad (48)$$

The value of a firm that does not receive any order is given by

$$V_{0t} = e^{- \ _e dt} E_t \frac{C_{e;t}}{C_{e;t+dt}} (\ _{t+dt} V_{1t+dt} + (1 - \ _{t+dt}) V_{0t+dt}). \quad (49)$$

Define $V_t = \ _t V_{1t} + (1 - \ _t) V_{0t}$ as the expected value of the firm. The firm has no incentive to produce lemons if and only if $V_{1t} \geq V_t^d$, or

$$\Phi dt \leq (1 - \ _t)\Phi dt + \ dt e^{- \ _e dt} \frac{C_{e;t}}{C_{e;t+dt}} V_{t+dt}. \quad (50)$$

In the limit $dt \to 0$, the incentive compatibility condition becomes $_t \Phi \leq \ V_t$.[15] Then the expected value of the firm is given by the present discounted value of all future profits as

$$V_t = \int_0^\infty e^{- \ _e s} \frac{C_{et}}{C_{es}} \Pi_s ds. \quad (51)$$

For simplicity, we assume Φ is big enough such that $_t = Y_t/\Phi < 1$ always holds. The average profitis then obtainedas $\Pi \ _t = (1 - \ _t) Y_t$. Then using $C_{ej} = \Pi_j$ and integratingthe right hand side of equation 51, we have

$$V_t = \frac{(1 - \ _t) Y_t}{[illegible] _e}. \quad (52)$$

[15] Under the incentive compatibility condition we can consider one-step deviations since V_{1t}, and V_{0t} are then optimal value functions.

The households' budget constraint changes to

$$C_t + I_t \le R_t u_t K_t + W_t N_t = \ _t Y_t: \tag{53}$$

Then the incentive constraint (50) becomes

$$_t\Phi \le \frac{(1 - \ _t) Y_t}{e}: \tag{54}$$

From the household budget constraint (53), we know that household utility increases with $_t$ and thus the incentive constraint (54) must be binding. Then equation (54) can be simplified as

$$_t = \frac{Y_t}{\Phi + Y_t} < 1; \tag{55}$$

where now $\equiv \frac{e}{}$. Similar to the baseline model, here firms also receive an information rent. However, the rent in the baseline is derived from hidden information while the rent here arises from hidden action. As indicated in equation (55), $_t$ is procyclical and hence the markup is countercyclical. When output is high, the total profit from production is high. Therefore the value of a good reputation is high and the opportunity cost of defaulting also increases. This then alleviates the moral hazard problem since high output dilutes information rent.

The cost minimization problem again yields the factor prices given by equation (20) and (21). Since households do not own firms, their budget constraint is modified as

$$C_t + \dot{K}_t = \ _t Y_t - \ (u_t) K_t: \tag{56}$$

The equilibrium system of equations is the same as in the baseline model except that equation (19) is replaced by equation (56). The steady state can be computed similarly. The steady state output is given by

$$Y = A^{\frac{1}{1 \ \alpha}} \left[\left(\frac{}{(1 + \ )} \right)^{\frac{\alpha}{1 \ \alpha}} \left(\frac{1 - }{1 - \frac{}{1+}} \right) \cdot \frac{1}{} \right]^{\frac{1}{1+\gamma}} \equiv Y(\ ); \tag{57}$$

and can be solved from

$$\bar{\Phi} \equiv \ \Phi \equiv \Psi(\ ) = \frac{1 - }{} \cdot Y(\ ): \tag{58}$$

Unlike in the baseline model, here the steady state equilibrium is unique as $Y(\ )$ is monotonic.[16] We summarize the result in the following lemma.

[16] Note that compared to equation (32), is missing from the numerator of the second bracket in equation (57).

Lemma 3 *If* $< \frac{1}{2}$*, a consistently standard calibrated value of* *, then the steady state equilibrium is unique for any* $\bar{\Phi} > 0$*.*

We can now study the possibility of self-fulfilling equilibria around the steady state. Since and $\bar{\Phi}$ form a one-to-one mapping, we will treat as a free parameter in characterizing the indeterminacy condition. We can then use equation (58) to back out the corresponding value of $\bar{\Phi}$. The following proposition specifies the condition under which self-fulfilling equilibria arises.

Proposition 3 *Let* $= 1 -$ *. Then indeterminacy emerges if and only if*

$$_{\min} < \quad < \min \quad \frac{1+}{} - 1; \; _H \quad \equiv \quad _{\max};$$

where $_{\min} \equiv \frac{(1+\ )(1+\ )}{(1+\ )(1-\ )+\ (1+\ )} - 1;$ *and* $_H$ *is the positive solution to* $A_1 \ ^2 - A_2 \ - A_3 = 0$*, where*

$$\begin{aligned} A_1 &\equiv s(1+\ )(2+\ \ +\ \ ) \\ A_2 &\equiv (1+\ )(1+\ \ ) - s[(1+\ )(1-\ )(1-\ ) + (1+\ )\ ] \\ A_3 &\equiv (1+\ )(1-\ )[s + (1-s)\ ]. \end{aligned}$$

Indeterminacy implies that the model exhibits multiple expectation-driven equilibria around the steady state. The steady state equilibrium is now unique however, which suggests that the continuum of equilibria implied by indeterminacy cannot be obtained in static models studied the earlier literature. So far, the condition to sustain indeterminacy is given in terms of and . The following corollary specifies the underlying condition in terms of $_e$, and Φ.

Corollary 4 *Indeterminacy emerges if and only if* $\frac{\Psi(1-\ _{\min})}{\Phi} < \frac{_e}{} < \frac{\Psi(1-\ _{\max})}{\Phi}$.

Given the other parameters, a decrease in $_e$ or an increase in increases the steady state . According to the above lemma, it makes indeterminacy less likely. The intuition is straightforward. A large means the opportunity cost of defaulting increases, as the firm becomes more likely to be excluded from future production. This alleviates the moral hazard problem, which is the source of indeterminacy. Similarly, a decrease of $_e$ means that the entrepreneurs become more patient. So the future profit flow from production is more valuable to them, which again increases the opportunity cost of producing lemons and thus alleviates the moral hazard problem.

where the left-hand side is the actual repayment from the final goods firms, and the right-hand side is the actual lending. Then the interest rate is given by

$$R_{ft} = \frac{1}{\int_0^{q_t} q dF(q) / \int_0^{q_t} dF(q)} = \frac{1}{E(q|q \leq q_t^*)} > 1; \tag{64}$$

where the denominator is the average success rate. The above equation says that the interest rate decreases with the average success rate.

The total production of final goods is

$$Y_t = \int_0^1 q_j a_{jt} x_{jt} dF(q) = \Phi \int_0^{q_t} a_{\min} q^{1-} \, dF(q). \tag{65}$$

where the second equality follows equation (61). The total production of intermediate goods is

$$X_t = \Phi \int_0^{q_t} \, dF(q). \tag{66}$$

Finally the intermediate goods are produced according to $X_t = A_t (u_t K_t) \;\; N_t^{1-}$, where $u_t K_t$ is the capital rented from the households. Combining equations (65) and (66) then yields

$$Y_t = \Gamma(q_t^*) A_t (u_t K_t) \;\; N_t^{1-} \;\; ; \tag{67}$$

where $\Gamma(q_t^*) = \int_0^{q_t} a_{\min} q^{1-} \, dF(q) \;\; / \int_0^{q_t} dF(q)$ depends on the threshold q_t^* and the distribution. The above equation then says that the measured TFP is obtained as

$$TFP_t = \frac{Y_t}{(u_t K_t) \;\; N_t^{1-}} = \Gamma(q_t^*) A_t. \tag{68}$$

Since $\Gamma'(q_t^*) = \frac{a_{\min} f(q_t) \int_0^{q_t} \left(q_t^{1 \;\; \tau} - q^{1 \;\; \tau}\right) dF(q)}{\left(\int_0^{q_t} \, dF(q)\right)^2} > 0$, the endogenous TFP increases with the threshold q_t^*. This is very intuitive: as the threshold increases, more firms with high productivity enter the credit market, making resource allocation more efficient. Equation (65) implies that q_t^* increases with Y_t, so we get the following lemma.

Lemma 4 *TFP is endogenous and increase in Y, i.e.,* $\frac{\partial \Gamma(q_t)}{\partial Y_t} > 0.$

We have therefore established that the endogenous TFP is procyclical. Notice that the procyclicality of endogenous TFP holds generally for continuous distributions. So without loss of generality, we now assume $F(q) = q$ for tractability. In turn, firm-level measured productivity $\frac{1}{q}$ follows a Pareto distribution with the shape parameter of , which is consistent with the findings of a large literature (see, e.g., Melitz (2003) and references therein). Under the assumption of a power distribution, combining equations (65) and (67) yields the aggregate output

$$Y_t = \frac{\quad}{- \quad +1} \quad a_{\min} \Phi^{-\frac{1 \quad \tau}{\eta}} \quad A_t u_t \, K_t \, N_t^{1-} \quad {}^{1+\frac{1 \quad \tau}{\eta}} : \tag{69}$$

The intuition is as follows. Here a lending externality kicks in because of adverse selection in the credit markets. Suppose that the total lending from financial intermediaries increases. This creates downward pressure on interest rate R_{ft}, which increases the cutoff q_t^* according to the definition in equation (62). Firms with a higher q have a smaller risk of default. A rise in the cutoff q_t^* therefore reduces the average default rate. If the rise is big enough, it can in turn stimulate more lending from the financial intermediaries. Since firms with higher q are also more productive on average, the increased efficiency in reallocating credit implies that resources are better allocated across firms. Notice that the aggregate output again exhibits increasing returns to scale. Equation (69) reveals that the degree of increasing returns to scale clearly depends on the adverse selection problem and decreases with and $:$ When $= \infty$, the firms' product quality is homogeneous. Hence there is no asymmetric information or adverse selection. If $= 1$, firms are equally productive in the sense their expected productivity is the same. It therefore does not matter how credits are allocated among firms. Given $< 1;$ a smaller implies that firms are more heterogenous, creating a larger asymmetric information problem. Similarly, given , a smaller implies that the productivity of firms deteriorates faster with respect to their default risk, making the adverse selection more damaging to resource allocation. We formally state this result in the following proposition.

Proposition 4 *The reduced-form aggregate production in our model exhibits increasing returns to scale if and only if there exists adverse selection, $i.e.;$ < 1 and $< \infty$.*

In an important contribution, Basu and Fernald (1997) document that increasing returns to scale exist in aggregate production but not at the micro level. In a recent paper, Liu and Wang (2014) show how credit constraints can generate endogenous variation in TFP, and hence aggregate increasing returns. In their model, the less productive firms are driven out of production. Different from Liu and Wang (2014), firms in our model do not suffer from credit constraints; the more productive firms in our model are driven out of production due to adverse selection.

As in the benchmark model, both the credit spread, $R_{ft} - 1$, and the expected default risk, $1 - E\,(q|q \leq q_t^*)$, are countercyclical. These predictions are consistent with the empirical regularities by Gilchrist and Zakrajšek (2012) and many others.

4.1 Indeterminacy

It is straightforward to show that $W_t = \frac{(1- \ )Y_t}{N_t}$ and $R_t = \frac{Y_t}{u_t K_t}$ respectively. Here $\ = \frac{+1-}{+1}$ and is constant instead of procyclical. Together with equations (5), (6), (7), (69), and (19), we can determine the seven variables, C_t, Y_t, N_t, u_t, K_t, W_t and R_t. The steady state can be obtained as in the baseline model. We can express the other variables in terms of the steady state . Since is unique, unlike in the baseline model, the steady state here is unique. We assume that Φ is large enough so that an interior solution to q^* is always guaranteed. The following proposition summarizes the conditions for indeterminacy in this extended model.

Proposition 5 *Given the power distribution, i.e., $F(q) = (q/q_{\max})$, (or equivalently, firm productivity conforms to a Pareto distribution), the steady state is unique. Moreover, the model is indeterminate if and only if*

$$_{\min} < \quad < \ _{\max} \tag{70}$$

where $\equiv \frac{1-}{}$, $_{\min} \equiv \frac{1}{\frac{1 \ \alpha}{1+\gamma} + \frac{\alpha}{1+\theta}} - 1$ *and* $_{\max} \equiv \frac{1}{} - 1.$

To better understand the proposition, we first consider how output responds to a fundamental shock, such as a change in A, the true TFP. Holding factor inputs constant, we have

$$1 + \tilde{} \equiv \frac{d \log Y_t}{d \log A} = (1 + \ ) \ \frac{1 +}{1 + \ - \ (1 + \ )} > 1, \tag{71}$$

The above equations show that adverse selection and variable capacity utilization can amplify the impact of a TFP shock on output. Let us define $1 + \tilde{}$ as the multiplier of adverse selection. Note that the necessary condition $\ > \ _{\min}$ can be written as

$$(1 + \tilde{})(1 - \ ) - 1 > \ . \tag{72}$$

The model will be indeterminate if the multiplier effect of adverse selection is sufficiently large. The restriction $\ < \ _{\max}$ is typically automatically satisfied. The restriction $\ < \frac{1}{} - 1$ simply requires that $\ (1 + \ ) < 1$, which is the condition needed to rule out explosive growth in the model.

Whether the model is indeterminate or not, equation (71) implies that the response of output to TFP shocks will be amplified. In addition, by Proposition 4, the economy is more likely to be indeterminate if is smaller. Our results are hence in the same spirit as those of Kurlat (2013) and Bigio (2014), showing that a dispersion in quality will strengthen the amplification effect of adverse selection.

Empirical Possibility of Indeterminacy To empirically evaluate the possibility of indeterminacy, we set the same values for , , , and as in Table 1.[17] We also have new parameters in this extended model, (;). We use two moments to pin them down and set and to match the steady state markup $\frac{+1-}{+1} = 0.9$. Basu and Fernald (1997) estimate aggregate increasing returns to scale in manufacturing to approximately 1.

Appendix

A Proofs

Proof of Lemma 1: The proof is straightforward. First, from the explicit form of $Y(\ )$, we can easily prove that $\Psi(\ ) \equiv \frac{1-}{} \cdot Y(\ )$ strictly increases with when $\in (0; \ ^*)$ but strictly decreases with when $\in (\ ^*; 1)$. Second, since $\Psi(0) < \bar{\Phi} < \Psi^* = \Psi(\ ^*)$, there exists a unique solution between zero and *, denoted by $\bar{\ }_L$, that solves $\Psi(\ ) = \bar{\Phi}$. Likewise, there also exists a unique solution between * and 1, denoted by $\bar{\ }_H$, that solves $\Psi(\ ) = \bar{\Phi}$.

Proof of Lemma 2: Denote by $'_1$ and $'_2$ the eigenvalues of matrix J so that we have $'_1 + '_2 =$Trace(J) and $'_1 '_2 =$Det(J). Then the model is indeterminate if the trace of J is negative and the determinant is positive. The trace and the determinant of J are

$$\frac{\text{Trace}(J)}{} = \frac{1+}{} \quad _1 - (1+\ )\ _1 + \ (1+\ )\ _2;$$

$$\frac{\text{Det}(J)}{2} = [(1+\ )\ _1 - 1 + \ _2] \quad \frac{1+}{} - 1 \ - \ \ _2;$$

respectively, where

$$_1 = \frac{a(1+\ )}{1+\ -b(1+\ )}; \text{ and } \ _2 = -\frac{b}{1+\ -b(1+\ )};$$

as defined in equation (36).

Substituting out $_1$ and $_2$ we obtain

$$\frac{\text{Trace}(J)}{} = \frac{1}{+1-(1+\ )b} \cdot \quad \frac{1+}{} - 1 - \quad a(1+\ ) - \ (1+\ )b$$

$$= - \frac{(1+\ )+(1+\ )(1-\ )}{1+\ -(1+\ )} \cdot \left[\frac{\frac{(1+\ )(1+\ )}{(1+\ )+(1+\ )(1-\ )} - \ (1+\ )}{+1-(1+\ )b}\right]$$

$$= - \frac{(1+\ )+(1+\ )(1-\ )}{1+\ -(1+\ )} \cdot \left[\frac{\frac{(1+\ )(1+\ )}{(1+\ )+(1+\ )(1-\ )} - 1 + \ ^2}{+1-(1+\ )b}\right]$$

Notice that $+1-(1+\ )b <$

Therefore Trace$(J) < 0$ if and only if $\ > \ _{\min}$: It remains for us to determine the condition under which Det$(J) > 0$. Note that Det(J) can be rewritten as

$$\frac{\mathrm{Det}\,(J)}{\ ^2} = \frac{1}{\ +1-(1+\ )b} \cdot \frac{1+\ }{\ } -1 \quad ((1+\ )\,[a(1+\ )-1]+\ b)+\ b$$

$$= \frac{1+\ }{(1+\ )b-(\ +1)} \quad (1+\ )(1-\ )-\frac{(1-\ )(1+\ )}{(1+\ -\ )}+(1+\ ) \qquad :$$

If $\ < \ _{\min}$, then we immediately have Det$(J) < 0$. Thus to guarantee that Det$(J) > 0$, we must have $\ > \ _{\min}$, which then implies that $(1+\ )b-(\ +1) > 0$. As a result, given that $\ > \ _{\min}$, Det$(J) > 0$ if and only if

$$(1+\ )(1-\ )-\frac{(1-\ )(1+\ )}{1+\ -\ }+(1+\ ) \qquad > 0;$$

which can be further simplified as

$$< \frac{(1+\ )(1-\ )}{\frac{(1-\ )(1+\ )}{1+\ -\ }+(1+\ )}:$$

Since $\ =1-\ $, the above inequality can be reformulated as

$$\Delta(\ ) \equiv \ ^2\ ^2+\ \ +\frac{(1-\ )\,(1+\ )}{(1+\ )} \quad -(1-\ )\,(1+\ -\ ) < 0:$$

Denote $\ \equiv \ +\frac{(1-\ )(1+\ )}{(1+\ )}$. Then det$(J) > 0$ if and only if $\ > \ _{\min}$ and

$$< \ _{\max} \equiv \frac{-\ +\ \sqrt{\ ^2+4\ ^2(1-\ )\,(1+\ -\ )}}{2\ ^2}:$$

It remains for us to prove that $\ _H=1-\ ^*$, where $\ ^*=\arg\max_{0\leq\ \leq 1}\Psi(\ )$. The first-order condition of $\log\Psi(\ )$ suggests

$$\frac{1}{1+\ }+\frac{2\ -1}{1-\ } \quad \frac{1}{\ } \ + \ \frac{1}{1+\ } \quad \frac{\ }{1+\ } \quad \frac{1}{1-\frac{\ }{1+\ }} \ ! -\frac{1}{1-\ }=0;$$

which is equivalent to

$$\Gamma(\ ) \equiv \ ^2\ ^2-\frac{(1-\ )\,(1+\ )}{1+\ }+\ \ +2\ ^2 \quad +\frac{(1-\ )\,(1+\ )}{1+\ }+(2\ -1)\,(1+\ ) \ =0:$$

Besides, we can easily verify that; for $\ \in(0;1)$, it always holds that

$$\frac{d^2}{d\ ^2}\,(\log\Psi(\ )) < 0:$$

Since $\ \equiv 1-\ $, we know that $\Delta\,(1-\ )=\Gamma\,(\ )$. Denote by $\ _1$ and $\ _2$ the solutions to $\Gamma(\ )=0$. Note that $\ _1+\ _2 > 0$, $\ _1\cdot\ _2 > 0$, and $\Gamma\,(0) > 0$, $\Gamma\,(1) > 0$. Therefore we know that $0 < \ _1 < 1 < \ _2$. Consequently we conclude that

$$^*=\ _1=1-\ _{\max}\in(0;1)\,.$$

Proof of Proposition 1: Notice that, by definition, $_{\max} = 1 - \ _{\min}$. Therefore we have $_{\min} = \ ^*$. Then by Lemma 2 we know that

1. If $\ < \ _{\min}$, then Trace$(J) < 0$, and Det$(J) < 0$.

2. If $\ \in (\ _{\min}; \ _{\max})$, then Trace$(J) < 0$, and Det$(J) > 0$.

3. If $\ > \ _{\max}$, then Trace$(J) > 0$, and Det$(J) < 0$.

Proof of Corollary 1: First, when adverse selection is severe enough, *i.e.*, $\bar{\Phi} = \ \Phi \geq \Psi_{\max}$, the economy collapses. The only equilibrium is the trivial case with $\ = 0$. Given that $\bar{\Phi} < \Psi_{\max}$; Lemma 1 implies that there are two solutions, which are denoted by $\bar{\ }_H; \bar{\ }_L$. It always holds that $\bar{\ }_L < \ ^* < \bar{\ }_H$. Then Lemma 2 immediately suggests that the steady state $\bar{\ }_L$ is always a saddle. Since $\Psi(\ )$ decreases with $\$ when $\ > \ ^*$, as shown in Proposition 1, indeterminacy emerges if and only if $\ \in (\ ^*; \ _{\max})$. Therefore the local dynamics around the steady state $\ = \bar{\ }_H$ exhibits indeterminacy if and only if $\Psi(\ _{\max}) < \bar{\Phi} < \Psi_{\max}$.

Proof of Corollary 2: Holding Φ constant, $\bar{\Phi}$ increases with , the proportion of firms producing lemon products. As is proved in Corollary 1, given $\bar{\Phi} < \Psi_{\max}$, indeterminacy emerges if and only if $\bar{\Phi} > \Psi(\ _{\max})$. Therefore the likelihood of indeterminacy increases with .

Proof of Proposition 2: As shown in Section 2, the dynamical system on $(C_t; K_t)$ is given by

$$\frac{\dot{C}_t}{C_t} = \frac{}{1+} \quad {}_t\frac{Y_t}{K_t} - \quad ; \tag{A.1}$$

$$\dot{K}_t = Y_t - \quad {}_0\frac{u_t^{1+}}{1+} \quad K_t - C_t. \tag{A.2}$$

where

$$u_t^{1+} = \frac{}{{}_0}\frac{{}_tY_t}{K_t}; \tag{A.3}$$

$$Y_t = Y(\ _t) \equiv \frac{t}{1 - \ _t} \quad \Phi; \tag{A.4}$$

and

$$(u_t) \equiv {}_0\frac{u_t^{1+}}{1+};$$

in which $^{0} = -(1 + \ )$ so that $u = 1$ at the steady state.

First, equation (A.3) implies

$$u_t = \left(\frac{{}_t Y_t}{{}^0 K_t} \right)^{\frac{1}{1+\theta}};$$

and thus we have

$$N_t^{1-} = \frac{Y_t}{A u_t K_t} = \frac{Y_t^{1-\frac{\alpha}{1+\theta}} \ {}_t^{-\frac{\alpha}{1+\theta}} K_t^{-\frac{\alpha\theta}{1+\theta}}}{A \ \frac{}{0}^{\frac{\alpha}{1+\theta}}}: \tag{A.5}$$

Substituting equation (A.5) into (5) yields

$$\left[\frac{Y_t^{1-\frac{\alpha}{1+\theta}} \ {}_t^{-\frac{\alpha}{1+\theta}} K_t^{-\frac{\alpha\theta}{1+\theta}}}{A \ \frac{}{0}^{\frac{\alpha}{1+\theta}}} \right]^{1+} = \frac{1}{C_t} \left(\frac{1 -}{} \ {}_t Y_t \right)^{1-};$$

which can be further simplified as

$$\frac{Y_t^{\left(1-\frac{\alpha}{1+\theta}\right)(1+\ )} \ {}_t^{-\frac{\alpha(1+\gamma)}{1+\theta}} K_t^{-\frac{\alpha\theta(1+\gamma)}{1+\theta}}}{A^{1+} \ \frac{}{0}^{\frac{\alpha(1+\gamma)}{1+\theta}}} = C_t^{-(1-\ )} \left(\frac{1-}{} \right)^{(1-\ )} {}_t^{1-} Y_t^{1-};$$

or equivalently,

$$C_t^{1-} = A^{1+} \ \frac{}{0}^{\frac{\alpha(1+\gamma)}{1+\theta}} \left(\frac{1-}{} \right)^{(1-\ )} {}_t^{1-\ +\frac{\alpha(1+\gamma)}{1+\theta}} Y_t^{1-\ -\left(1-\frac{\alpha}{1+\theta}\right)(1+\ )} K_t^{\frac{\alpha\theta(1+\gamma)}{1+\theta}}: \tag{A.6}$$

Substituting equation (A.4) into (A.6) yields

$$C_t = C(\ {}_t; K_t) = f_0 \cdot g(\ {}_t) \cdot h(K_t); \tag{A.7}$$

where $f_0 = A^{\frac{1+\gamma}{1\ \alpha}} \ \frac{}{0}^{\frac{\alpha(1+\gamma)}{(1+\theta)(1\ \alpha)}} \ \frac{1-}{}$, $h(K_t) = K_t^{\frac{\alpha\theta(1+\gamma)}{(1+\theta)(1\ \alpha)}}$, and

$$g(\ {}_t) = \left[{}_t^{1-\ +\frac{\alpha(1+\gamma)}{1+\theta}} Y(\ {}_t)^{1-\ -\left(1-\frac{\alpha}{1+\theta}\right)(1+\ )} \right]^{\frac{1}{1\ \alpha}}:$$

In turn, differentiating both sides of equation (A.7) yields

$$C_t^{1-} = A^{1+} \ \frac{}{0}^{\frac{\alpha(1+\gamma)}{1+\theta}} \left(\frac{1-}{} \right)^{(1-\ )} {}_t^{1-\ +\frac{\alpha(1+\gamma)}{1+\theta}} Y_t^{1-\ -\left(1-\frac{\alpha}{1+\theta}\right)(1+\ )} K_t^{\frac{\alpha\theta(1+\gamma)}{1+\theta}};$$

which immediately implies

$$(1- \quad)\frac{\dot{C}_t}{C_t} = 1- \quad + \frac{(1+ \quad)}{1+} \quad \frac{\dot{t}}{t} + 1- \quad - \quad 1 - \frac{}{1+} \quad (1+ \quad) \frac{\dot{Y}_t}{Y_t} + \frac{(1+ \quad)}{1+} \quad \frac{\dot{K}_t}{K_t}$$

$$= 1- \quad + \frac{(1+ \quad)}{1+} + 1- \quad - \quad 1 - \frac{}{1+} \quad (1+ \quad) \frac{Y'(\ _t)\ _t}{Y(\ _t)} \quad \frac{\dot{t}}{t} + \frac{(1+ \quad)}{1+} \quad \frac{\dot{K}_t}{K_t}$$

$$= 1- \quad + \frac{(1+ \quad)}{1+} - \quad 1 - \frac{}{1+} \quad (1+ \quad) - (1- \quad) \quad \frac{1}{1- \ _t} \quad \frac{\dot{t}}{t} + \frac{(1+ \quad)}{1+} \quad \frac{\dot{K}_t}{K_t}$$

$$= 1- \quad + \frac{(1+ \quad)}{1+} \quad \frac{_{\max} - \ _t}{1- \ _t} \quad \frac{\dot{t}}{t} + \frac{(1+ \quad)}{1+} \quad \frac{\dot{K}_t}{K_t} \qquad \text{(A.8)}$$

Additionally, we have

$$u_t = \quad \frac{}{_0} \frac{_t Y(\ _t)}{K_t}^{\frac{1}{1+\theta}} \equiv u(K_t; \ _t): \qquad \text{(A.9)}$$

In the end, substituting equation (A.7) and (A.9) into (A.1) and (A.2) yields

$$1- \quad + \frac{(1+ \quad)}{1+} \quad \frac{_{\max} - \ _t}{1- \ _t} \quad \frac{\dot{t}}{t} + \frac{(1+ \quad)}{1+} \quad \frac{\dot{K}_t}{K_t} = (1- \quad) \quad \frac{}{1+} \quad _t \frac{Y(\ _t)}{K_t} - \quad ;$$

$$\dot{K}_t = 1 - \frac{_t}{1+} \quad Y(\ _t) - C(\ _t; K_t);$$

the desired autonomous dynamical system in Proposition 2.

Proof of Corollary 3: We can easily verify that $g(0) = g(1) = 0$, $g''(\ ) < 0$, and $g'(\ _{\max}) = 0$, where $_{\max} = 1 - \ _{\min}$, and $_{\min}$ is defined in Lemma 2. Therefore we have $_{\max} = \arg\max g(\ ):$ It then follows from equation (43) that C_t is a hump-shaped function of $_t$ for a given level of K_t. Then we immediately obtain the results in Lemma 3.

Proof of Lemma 3: Notice that $\Psi(\ ) = \frac{1-}{} \cdot Y(\ ) \propto (1- \quad)^{\frac{2\alpha \ 1}{1 \ \alpha}}$. When $\ < \frac{1}{2}$, we know that $(1- \quad)^{\frac{2\alpha \ 1}{1 \ \alpha}}$ is decreasing in . It is easy to check that $\lim_{\ \to 0} \Psi(\ ) = \infty$ and $\lim_{\ \to 1} \Psi(\ ) = 0$. Hence equation (58) uniquely pins down the steady state for any $\bar{\Phi} > 0$.

Proof of Proposition 3: The dynamical system with reputation is given by

$$
\begin{aligned}
N_t &= \frac{1}{C_t}(1- \quad)\; {}_t\frac{Y_t}{N_t};\\
\frac{\dot{C}_t}{C_t} &= \quad {}_t\frac{Y_t}{K_t} - \quad (u_t) - \quad ;\\
{}_t\frac{Y_t}{u_t K_t} &= {}^{0}u_t;\\
C_t + \dot{K}_t + C_t^e &= Y_t - \quad (u_t)\, K_t;\\
Y_t &= A\,(u_t K_t) \quad N_t^{1-} \quad ;\\
{}_t &= \frac{Y_t}{\Phi + Y_t};\\
C_t^e &= (1- \quad {}_t)\, Y_t;
\end{aligned}
$$

where $\equiv \frac{\ \ }{e}$. Denote $s \equiv 1 - \frac{\ \ }{1+}$. Then some of the key ratios in the steady state can be obtained as

$$
\begin{aligned}
k_y &= \frac{K}{Y} = \frac{\ \ }{(1+ \quad)};\\
c_y &= \frac{C}{Y} = s \quad = \quad 1 - \frac{\ \ }{1+} \quad ;\\
N &= \frac{(1- \quad)}{c_y} \cdot \frac{1}{\ \ }^{\frac{1}{1+\gamma}} = \left[\left(\frac{1-}{1-\frac{\ \ }{1+}} \right) \cdot \frac{1}{\ \ } \right]^{\frac{1}{1+\gamma}};\\
Y &= A^{\frac{1}{1\ \alpha}}\,(k_y)^{\frac{\alpha}{1\ \alpha}}\, N = A^{\frac{1}{1\ \alpha}} \left(\frac{\ \ }{(1+ \quad)} \right)^{\frac{\alpha}{1\ \alpha}} \left[\left(\frac{1-}{1-\frac{\ \ }{1+}} \right) \cdot \frac{1}{\ \ } \right]^{\frac{1}{1+\gamma}}: \qquad \text{(A.10)}
\end{aligned}
$$

We can use equation (58) to solve for the steady state and use equation (A.10) to obtain the steady state Y. Consumption and capital can then be computed from $C = c_y Y$ and $K = k_y Y$, respectively. The log-linearization of the system of equilibrium equations is given by:

$$
\begin{aligned}
0 &= \hat{\ }_t + \hat{y}_t - (1+ \quad)\,\hat{n}_t - \hat{c}_t;\\
\dot{c}_t &= \quad \hat{\ }_t + \hat{y}_t - \hat{k}_t \quad ;\\
\hat{y}_t &= \quad \hat{u}_t + \hat{k}_t \quad + (1- \quad)\hat{n}_t;\\
\hat{u}_t &= \frac{1}{1+}(\hat{\ }_t + \hat{y}_t - \hat{k}_t);\\
\dot{k}_t &= \frac{s}{k_y} \quad \hat{\ }_t + \hat{y}_t - \hat{k}_t \quad - \quad \frac{c_y}{k_y} \quad \hat{c}_t - \hat{k}_t \quad ;\\
\hat{\ }_t &= (1- \quad)\,\hat{y}_t \equiv \quad \hat{y}_t:
\end{aligned}
$$

As in the baseline model, we can substitute $\hat{u}_t$ and $\hat{\ }_t$ to obtain a reduced form of output in terms of capital and labor as

$$\hat{y}_t = \frac{\ \hat{k}_t + (1+\ )(1-\ )\hat{n}_t}{1+\ \ -(1+\ )} \equiv a\hat{k}_t + b\hat{n}_t;$$

where $a \equiv \frac{\ }{1+\ -(1+\ )}$ and $b \equiv \frac{(1+\ )(1-\ )}{1+\ -(1+\ )}$. We assume $\ < \frac{1+}{\ } - 1$, which is a reasonable restriction under standard calibrations, so that $a > 0$ and $b > 0$. Finally $\hat{n}_t$ can be expressed as a function of $\hat{y}_t$ and $\hat{c}_t$, and thus we have

$$\hat{y}_t = \frac{a(1+\ )}{1+\ \ -b(1+\ )}\hat{k}_t - \frac{b}{1+\ \ -b(1+\ )}\hat{c}_t \equiv \ _1\hat{k}_t + \ _2\hat{c}_t;$$

where $\ _1 \equiv \frac{a(1+\ )}{1+\ -b(1+\ )}$ and $\ _2 \equiv -\frac{b}{1+\ -b(1+\ )}$. Consequently the local dynamics is characterized by the following differential equations:

$$\begin{bmatrix} \dot{k}_t \\ \dot{c}_t \end{bmatrix} = \begin{bmatrix} \frac{1+}{\ }\ s\ (1+\ )\ _1 & \frac{1+}{\ }\ [s\ (1+\ )\ _2 - (1-s\ )] \\ [(1+\ )\ _1 - 1] & (1+\ )\ _2 \end{bmatrix} \begin{bmatrix} \hat{k}_t \\ \hat{c}_t \end{bmatrix};$$

$$\equiv J \begin{bmatrix} \hat{k}_t \\ \hat{c}_t \end{bmatrix};$$

where $s \equiv 1 - \frac{\ }{1+}$, $c_y = s\ $, $\ = \ = \ $. The local dynamics around the steady state is determined by the roots of J. Notice that the trace and the determinant of J are

$$\frac{\text{Trace}(J)}{\ } = \frac{1+\ }{\ }\ s(1+\ )\ _1 + \ (1+\ )\ _2 < 0;$$

$$\frac{\text{Det}(J)}{\ ^2\ \frac{1+}{\ }} = s\ (1+\ )\ _2 + (1-s\ )(1+\ )\ _1 - (1-s\ ) > 0.$$

Similar to the analysis of the indeterminacy for our baseline model, here Trace$(J) < 0$ if and only if $\ > \ _{\min} \equiv \frac{(1+\ )(1+\ )}{(1+\ )(1-\ )+\ (1+\ )} - 1$. Given that $\ > \ _{\min}$, some algebraic manipulation shows that Det$(J) > 0$ if and only if $\ < \frac{1+}{\ } - 1$, and

$$A_1\ ^2 - A_2\ \ - A_3 < 0;$$

where

$$A_1 \equiv s(1+\ )(2+\ \ +\ \ \ ) > 0$$

$$A_2 \equiv (1+\ )(1+\ \ \ ) - s[(1+\ )(1-\ )(1-\ ) + (1+\ )\ \ ]$$

$$A_3 \equiv (1+\ )(1-\ )[s+\ +(1-s)\ \ ] > 0.$$

Therefore $A_1\ ^2 - A_2\ \ - A_3 < 0$ if and only if $\ < \ _H$, where $\ _H$ is the positive solution to $A_1\ ^2 - A_2\ \ - A_3 = 0$.

Proof of Corollary 4: Combining Lemma 3 and Proposition 2 immediately yields the desired result.

Proof of Lemma 4: First, using the Implicit Function Theorem, equation (67) suggests that $\frac{\partial q}{\partial Y} > 0$. Second, since $TFP = \Gamma(q^*)A$, it is obvious that $\frac{\partial TFP}{\partial q} > 0$. Then using the chain rule gives $\frac{\partial TFP}{\partial Y} = \frac{\partial TFP}{\partial q} \quad \frac{\partial q}{\partial Y} \quad > 0$.

Proof of Proposition 4: We immediately reach the proposition by observing equation (69).

Proof of Proposition 5: First, given the power distribution, *i.e.*, $F(q) = (q/q_{\max})$, we can analytically obtain the dynamical system, and then easily verify the uniqueness of the steady state. It remains for us to pin down the indeterminacy region. To establish the conditions for indeterminacy, we first log-linearize the equilibrium equations. Substituting $\hat{u}_t$ from the log-linearized equation (24), we obtain

$$\hat{y}_t = a\hat{k}_t + b\hat{n}_t,$$

where $a = \frac{(1+ \ )}{1+ \ - \ (1+ \ )}$ and $b = \frac{(1+ \ )(1- \ )(1+ \ )}{1+ \ - \ (1+ \ )}$. Finally, expressing $\hat{n}_t$ from the log-linearized equation (22), we obtain

$$\hat{y}_t = \ _1\hat{k}_t + \ _2\hat{c}_t,$$

where $_1 \equiv \frac{a(1+ \ )}{1+ \ -b}$ and $_2 \equiv -\frac{a}{1+ \ -b}$. We hence obtain a two-dimensional system of differential equations

$$\begin{bmatrix} \dot{k}_t \\ \dot{c}_t \end{bmatrix} = \begin{bmatrix} \frac{1+}{} - 1 \quad _1 & \frac{1+}{} \quad (\ _2 - 1) + 1 - \ _2 \\ (\ _1 - 1) & \ _2 \end{bmatrix} \begin{bmatrix} \hat{k}_t \\ \hat{c}_t \end{bmatrix} \equiv J \begin{bmatrix} \hat{k}_t \\ \hat{c}_t \end{bmatrix}$$

where $=$ = . The local dynamics around the steady state is determined by the roots of J. The trace and the determinant of J are

$$\frac{\text{Trace}\,(J)}{} = \frac{1+}{} - 1 \quad _1 + \quad _2 = \frac{\frac{1+}{} - 1 \quad (1+ \quad)a - \quad b}{1+ \quad -b};$$

$$\frac{\det(J)}{2} = \frac{1+}{} - 1 \quad (\ _1 - 1 + \ _2) = \frac{1+}{} - 1 \quad \frac{(1+ \quad)(a-1)}{1+ \quad -b} \ .$$

Indeterminacy arises if Trace$(J) < 0$ and det$(J) > 0$. Under the assumption $a < 1$, or $(1+ \quad) < 1$, Det$(J) > 0$ is equivalent to $1 + \quad - b$, or $\quad > \ _{\min} \equiv \frac{1}{\frac{1 \ \alpha}{1+\gamma} + \frac{\alpha}{1+\theta}} \quad - 1$. Then

Trace(J) $<$ 0 requires $\frac{1+}{} - 1 \ (1 + \ ) a > \ b$. Rearranging terms yields the requirement, $\frac{(1+ \ )}{1+} < \frac{1}{\frac{1 \ \alpha}{1+\gamma} + \frac{\alpha}{1+\theta}}$. Recall that $\ = \frac{1-}{+}$, or $\frac{(1+ \ )}{1+} = \frac{}{+}$, so that this requirement is automatically satisfied.

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