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[illegible] δ [illegible]

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[illegible] δ [illegible]

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[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

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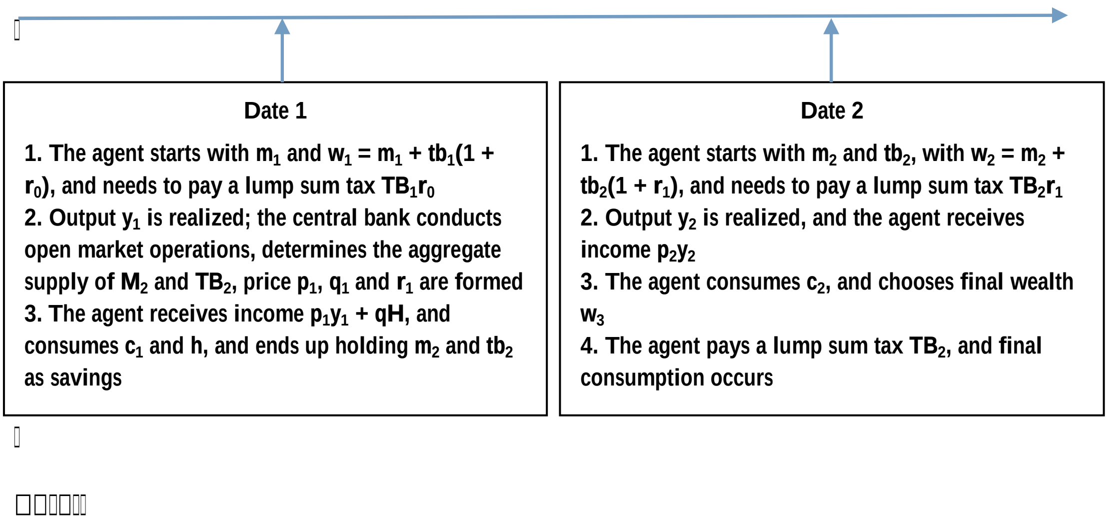


[illegible]

[illegible]

$$\max_{c_1,h_1,m_2,tb_2} u(c_1,h) + \beta U(m_2,tb_2)$$

$$\text{s.t.} \quad p_1c_1 + qh + m_2 + tb_2 \le p_1y_1 + qH + nw_1$$

$$p_1c_1 \le m_1 .$$

$$U(m_2,tb_2) = E\left[\max_{c_2,w_3}[v(c_2) + \alpha v(c_3)]\right]$$

$$\text{s.t.} \quad p_2c_2 + w_3 \le p_2y_2 + nw_2$$

$$p_2c_2 \le m_2$$

$$c_3 = nw_3 / p_2 = (w_3 - TB_2) / p_2$$

$$nw_2 = m_2 + tb_2(1+r_1) - TB_2 r$$

$$y_2 = f((TB_2 + MBS) / p_1, \varepsilon)$$

$$\mathrm{Max}_{M_2}\left\{u(c_1,h_1)+\beta U(M_2,TB_2)-\psi(MBS/TB_2)\right\}$$
$$\text{s.t.}\quad E[p_2/p_1]\geq\underline{\pi}$$

[illegible]

[illegible]

(i) [illegible]

(ii) [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

$$u(c_1, h_1) = \ln c_1 + \ln h_1$$
$$v(c_2) = \ln c_2, v(c_3) = \ln c_3.$$

$p_2 = M_2 / y_2.$

$$m_2 < nw_3 / \alpha \text{ with } nw_3 = M_2 + (tb_2 - TB_2)(1 + r_1)$$

[illegible]

$$U(m_2, tb_2) = (1+\alpha)\ln(f) + \ln(m_2 / M_2) + \alpha\ln(nw_3 / M_2);$$

[illegible]

$$m_2 > nw_3 / \alpha \text{ with } nw_3 = \frac{\alpha}{1+\alpha}\left[M_2 + m_2 + (tb_2 - TB_2)(1+r_1)\right]$$

[illegible]

$$U(m_2, tb_2) = (1+\alpha)\ln(f) + (2+\alpha)\ln(nw_3 / M_2) - \ln(\alpha).$$

[illegible] ■

[illegible]

[illegible] $U(M_2, TB_2) = (1+\alpha)\ln(f)$ [illegible]

[illegible]

[illegible]

$$p_1 = \frac{M_2}{\beta y_1}, \quad q = \frac{M_2}{\beta H}, \quad 1 + r_1 = \frac{1}{\alpha}$$

[illegible]

$$p_1 = \frac{M_1}{y_1}, \quad q = \frac{M_2}{\beta H}, \quad 1 + r_1 = \frac{1}{\alpha}$$

[illegible] ■

[illegible]

[illegible] $U_M(m_2 = M_2,\ tb_2 = TB_2)$ [illegible] $U_{TB}(m_2 = M_2,\ tb_2 = TB_2)$ [illegible]

$$
\begin{aligned}
U(m_2, tb_2) &= (1+\alpha)\ln(f) + \ln(m_2 / M_2) + \alpha \ln(nw_3 / p_2) \\
\text{with } nw_3 &= M_2 + (tb_2 - TB_2)(1+r_1).
\end{aligned}
$$

[illegible]

$$
\begin{aligned}
& U_m(m_2 = M_2, tb_2 = TB_2) = 1/M_2 \\
& U_{tb}(m_2 = M_2, tb_2 = TB_2) = \alpha(1+r_1)/M_2 \\
& U_M(m_2 = M_2, tb_2 = TB_2) = \frac{1+\alpha}{f} f_M \\
& U_{TB}(m_2 = M_2, tb_2 = TB_2) = \frac{1+\alpha}{f} f_{TB} = -\frac{1+\alpha}{f} f_M .
\end{aligned}
$$

[illegible]

[illegible]

[illegible]

[illegible]

$$
\begin{aligned}
V &= u(y_1, H) + \beta U(M_2, TB_2) - \psi(MBS/TB_2) \\
&= \ln(y_1) + \ln(H) + \beta(1+\alpha)\ln(f) - \psi(MBS/TB_2).
\end{aligned}
$$

[illegible] $MBS = QH = qH / r_1 = \alpha M_2 / \beta(1-\alpha)$ [illegible] $L = \left(TB_2 + \delta MBS\right) / p_1$ [illegible]

$$\max_{M_2} V(M_2) \equiv \beta(1+\alpha)\ln(f(L)) - \psi(MBS/TB_2) + C$$

$$= \begin{cases} \beta(1+\alpha)\ln f\left(\dfrac{[\beta NW + (\delta\alpha/(1-\alpha) - \beta)M_2]y_1}{M_2}\right) - \psi\left(\dfrac{\alpha M_2}{\beta(1-\alpha)(NW-M_2)}\right) + C \text{ if } M_2 < M_1\beta \\ \beta(1+\alpha)\ln f\left(\dfrac{[\beta NW + (\delta\alpha/(1-\alpha) - \beta)M_2]y_1}{\beta M_1}\right) - \psi\left(\dfrac{\alpha M_2}{\beta(1-\alpha)(NW-M_2)}\right) + C \text{ if } M_2 > M_1\beta. \end{cases}$$

[illegible]

[illegible]

[illegible] α [illegible] α [illegible]

[illegible] ■

[illegible]

[illegible]

[illegible]

[illegible] ■

[illegible]

[illegible]

$$M_\pi \equiv \frac{A\pi\beta NW}{\beta - A\pi(\delta\alpha/(1-\alpha)-\beta)}.$$

$$\psi\left(\frac{\alpha M_2}{\beta(1-\alpha)(NW-M_2)}\right) = \gamma\left[\ln NW - \ln(NW-M_2)\right]$$

$$M_\psi \equiv \frac{\beta(1+\alpha)(\delta\alpha/(1-\alpha)-\beta)-\gamma\beta}{\beta(1+\alpha)(\delta\alpha/(1-\alpha)-\beta)+\gamma(\delta\alpha/(1-\alpha)-\beta)}NW.$$

$M_\psi > M_\pi$ … $M_\psi > M_1\beta$ … $M_2 > M_1\beta$

■

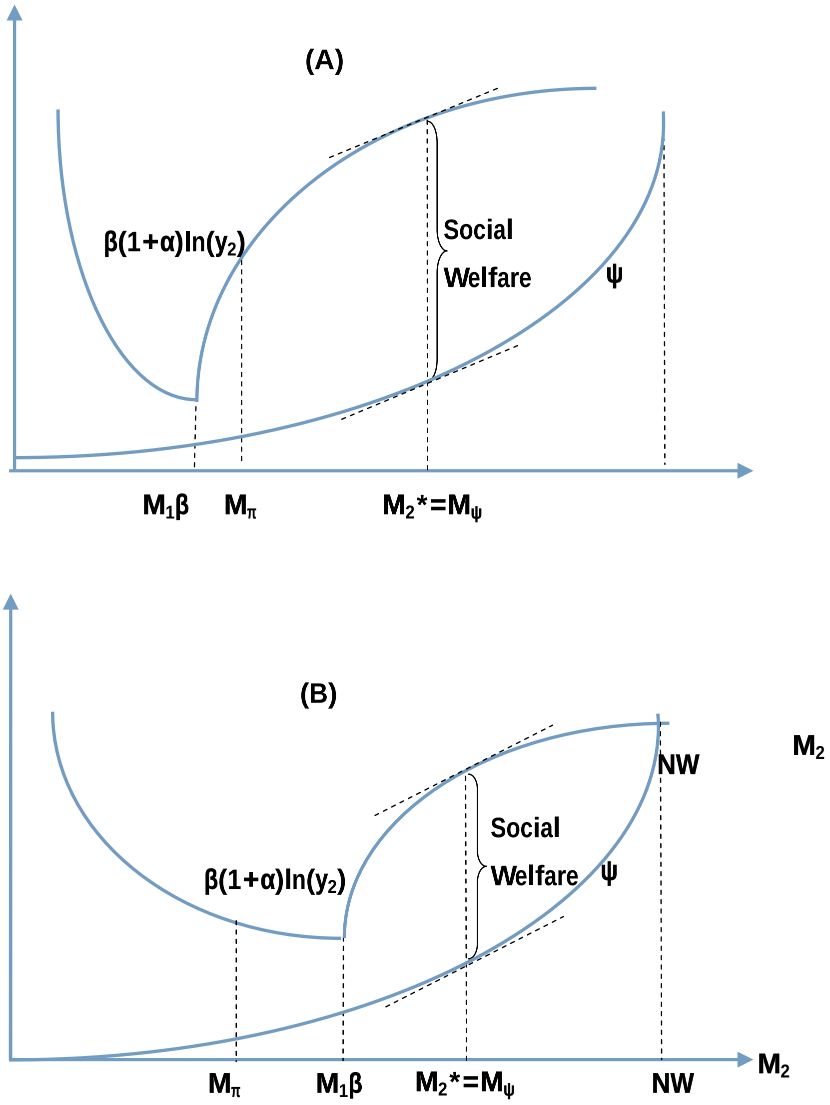
$M_\psi > M_\pi > M_1\beta$ … $M_\psi > M_1\beta > M_\pi$ … $M_1\beta > M_\psi > M_\pi$

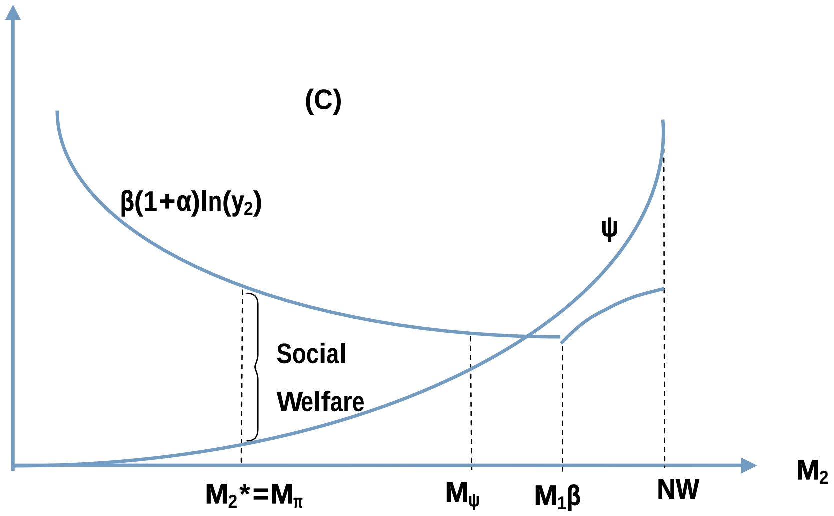
■

$M_\psi > M_\pi > M_1\beta$

$M_\psi > M_1\beta > M_\pi$

$M_1\beta > M_\psi > M_\pi$




(C)
β(1+α)ln(y2)
ψ
Social
Welfare
M2*=Mπ
Mψ
M1β
NW
M2

[illegible]

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[illegible]

[illegible]

$$\sigma_b = \{TB_t(\eta^t)\}_{t=1}^{\infty}$$ [illegible]

[illegible]

[illegible]

[illegible]

$$\max E\left[\sum_{t=1}^{\infty} \beta^t u(c_t, h_t)\right]$$

$$\text{s.t.} \quad \begin{aligned} & p_t c_t + q_t h_t + m_{t+1} + tb_{t+1} \le p_t y_t + q_t H + w_t - TB_t r_{t-1} \\ & p_t c_t \le m_t \end{aligned}$$ [illegible]

[illegible] $\{p_t, q_t, r_t\}_{t=1}^{\infty}$ [illegible] $\{c_t, h_t, m_{t+1}, tb_{t+1}\}_{t=1}^{\infty}$ [illegible]

[illegible] $\{p_t, q_t, r_t\}_{t=1}^{\infty}$ [illegible] $\{c_t, h_t, m_{t+1}, tb_{t+1}\}_{t=1}^{\infty}$ [illegible] $E\left[\sum_{t=1}^{\infty} \beta^t u(c_t, h_t)\right]$ [illegible]

[illegible]

[illegible]

[illegible] $p_{t+1}y_{t+1} < m_t$ [illegible]

$$\max_{c_{t+1},h_{t+1}} u(c_{t+1}, h_{t+1})$$

$$\text{s.t.}\quad \begin{aligned} & p_{t+1}c_{t+1} + q_{t+1}h_{t+1} + m_{t+2} + tb_{t+2} = p_{t+1}y_{t+1} + q_{t+1}H + w_{t+1} - TB_{t+1}r_t \\ & w_{t+1} = m_{t+1} + tb_{t+1}(1+r_t) \end{aligned}$$

[illegible] $c^{NB}_{t+1}(w_{t+1})$ [illegible] $h^{NB}_{t+1}(w_{t+1})$ [illegible]

[illegible]

$$\begin{aligned} X^{NB}_{t+1} &= u_c(c^{NB}_{t+1}, h^{NB}_{t+1})\frac{dc^{NB}_{t+1}}{dw_{t+1}}\frac{\partial w_{t+1}}{\partial m_{t+1}} + u_h(c^{NB}_{t+1}, h^{NB}_{t+1})\frac{dh^{NB}_{t+1}}{dw_{t+1}}\frac{\partial w_{t+1}}{\partial m_{t+1}} \\ &= u_c(y_{t+1}, H)c^{NB}_{w,t+1} + u_h(y_{t+1}, H)h^{NB}_{w,t+1} \end{aligned}$$

[illegible] $c^{NB}_{t+1}(w_{t+1}) = y_{t+1}$ [illegible] $h^{NB}_{t+1}(w_{t+1}) = H$ [illegible]

[illegible]

$$\begin{aligned} Z^{NB}_{t+1} &= u_c(c^{NB}_{t+1}, h^{NB}_{t+1})\frac{dc^{NB}_{t+1}}{dw_{t+1}}\frac{\partial w_{t+1}}{\partial tb_{t+1}} + u_h(c^{NB}_{t+1}, h^{NB}_{t+1})\frac{dh^{NB}_{t+1}}{dw_{t+1}}\frac{\partial w_{t+1}}{\partial tb_{t+1}} \\ &= u_c(y_{t+1}, H)c^{NB}_{w,t+1}(1+r_t) + u_h(y_{t+1}, H)h^{NB}_{w,t+1}(1+r_t) \\ &= X^{NB}_{t+1}(1+r_t) \end{aligned}$$

[illegible] $c^{NB}_{t+1}(w_{t+1}) = y_{t+1}$ [illegible] $h^{NB}_{t+1}(w_{t+1}) = H$ [illegible]

[illegible]

$$\begin{aligned} & u_c(y_{t+1}, H)/p_{t+1} = u_h(y_{t+1}, H)/q_{t+1} \\ & p_{t+1}c^{NB}_{w,t+1} + q_{t+1}h^{NB}_{w,t+1} = 1 \end{aligned}$$

$$u_c(y_{t+1},H)c^{NB}_{w,t+1} + u_h(y_{t+1},H)h^{NB}_{w,t+1} = u_c(y_{t+1},H)/p_{t+1} = u_h(y_{t+1},H)/q_{t+1}$$

X^{NB}_{t+1} and Z^{NB}_{t+1}

$$X^{NB}_{t+1} = u_c(y_{t+1},H)/p_{t+1}$$
$$Z^{NB}_{t+1} = u_c(y_{t+1},H)(1+r_t)/p_{t+1} = u_h(y_{t+1},H)(1+r_t)/q_{t+1}.$$

$$X_{t+1} = E_B[X^{B}_{t+1}]\Pr(B) + E_{NB}[X^{NB}_{t+1}]\Pr(NB)$$
$$Z_{t+1} = E_B[Z^{B}_{t+1}]\Pr(B) + E_{NB}[Z^{NB}_{t+1}]\Pr(NB)$$

$u(c_t,h_t) + \beta X_{t+1}m_{t+1} + \beta Z_{t+1}tb_{t+1}$

$$\max\left[u(c_t,h_t) + \beta X_{t+1}m_{t+1} + \beta Z_{t+1}tb_{t+1}\right]$$
$$\text{s.t.}\quad p_t c_t + q_t h_t + m_{t+1} + tb_{t+1} \le p_t y_t + q_t H + w_t - TB_t r_{t-1}$$
$$p_t c_t \le m_t$$

$$V[\sigma] = E\left[\sum_{t=1}^{\infty} \beta^{t} \left(u(c_t, h_t) - \psi(MBS_t / TB_{t+1})\right)\right]$$

1.

2. $\{c_t, h_t, m_{t+1}, tb_{t+1}\}_{t=\tau}^{\infty}$

$$X[\sigma] = E[u_c(c_1, h_1) / p_1]$$
$$Z[\sigma] = E[u_h(c_1, h_1)(1 + r_0) / q_1]$$

$$(V(M), X(M), Z(M)) = \{(V[\sigma], X[\sigma], Z[\sigma]) \mid \sigma \text{ is a PPE for the economy } \Phi(M)\}$$

$$
\begin{aligned}
&c(M, M'(M, y), y) = y \\
&h(M, M'(M, y), y) = H \\
&m'(M, M'(M, y), y) = M' = NW - TB'(M, y) \\
&tb'(M, M'(M, y), y) = TB'(M, y)
\end{aligned}
$$

$$
\begin{aligned}
&V(M, y) = u(y, H) - \psi(MBS / TB') + \beta E_{y'}\left[V(M', y')\right] \\
&\text{s.t.} \quad y' = f\left(\frac{TB' + \delta MBS}{p}, \varepsilon'\right)
\end{aligned}
$$

$$
X(M, y) = \begin{cases} u_c(y, H) / p & \text{if} \quad u_c(y, H) / p > u_h(y, H) / q \\ u_c(y, H) c_w^{NB} + u_h(y, H) h_w^{NB} & \text{if} \quad py < M \end{cases}
$$

$$
Z(M, y) = \begin{cases} u_h(y, H)(1 + r_0) / q & \text{if} \quad u_c(y, H) / p > u_h(y, H) / q \\ u_c(y, H)(1 + r_0) / p = u_h(y, H)(1 + r_0) / q & \text{if} \quad py < M \end{cases}
$$

$$
\begin{aligned}
&u_h(y, H) / q = \beta E_{y'}[X(M', y')] \\
&\text{s.t.} \quad y' = f\left(\frac{TB' + \delta MBS}{p}, \varepsilon'\right)
\end{aligned}
$$

$$
\begin{aligned}
&u_h(y, H) / q = \beta E_{y'}[Z(M', y')] \\
&\text{s.t.} \quad y' = f\left(\frac{TB' + \delta MBS}{p}, \varepsilon'\right)
\end{aligned}
$$

$$p = \begin{cases} M / y = (NW - TB) / y & \text{if} \quad u_c(y, H) / p > u_h(y, H) / q \\ q u_c(y, H) / u_h(y, H) & \text{if} \quad py < M \end{cases}$$

$$M'(M, y) = \arg\max_{M'} \left\{ u(y, H) - \psi(MBS / TB') + \beta E_{y'} \left[V(M', y') \right] \right\}$$

$$\text{s.t.} \qquad y' = f\left(\frac{TB' + \delta MBS}{p}, \varepsilon' \right)$$

■

$Y = [\underline{y}, \bar{y}]$

$$py = \begin{cases} M = NW - TB & \text{if} \quad M = py < qH \\ qH & \text{if} \quad py < M \end{cases}$$

[illegible]

$$X(M,y)=\begin{cases}1/py=1/M & \text{if} \quad M=py<qH\\ 1/py=1/qH & \text{if} \quad py<M\end{cases}$$

[illegible]

$$Z(M,y)=\begin{cases}(1+r_0)/qH & \text{if} \quad M=py<qH\\ (1+r_0)/py=(1+r_0)/qH & \text{if} \quad py<M\end{cases}$$

[illegible]

$$1/qH=\beta E\left[X(M',y')\right]$$

[illegible]

$$1/qH=\beta E\left[Z(M',y')\right]$$

[illegible]

$$V(M,y)=\max_{M'} u(y,H)-\psi(MBS/TB')+\beta E\left[V(M',y')\right]$$
$$\text{s.t.} \quad y'=f(L,\varepsilon)=f((TB'+\delta MBS)/p,\varepsilon), \quad p'/p\geq\underline{\pi}$$

[illegible]

[illegible] $r\approx g_q+(1-\beta)$ [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

98 ( ) 13.33903 ( ) 3.[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

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[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

$$
\begin{aligned}
&v_c(c_2) - \lambda_1 p_2 - \lambda_2 p_2 = 0\\
&\alpha v_c(nw_3 / p_2) / p_2 - \lambda_1 = 0\\
&p_2 c_2 + w_3 \le p_2 y_2 + nw_2\\
&p_2 c_2 \le m_2.
\end{aligned}
$$

[illegible]

$$
\begin{aligned}
&v_c(y_2) - \alpha v_w(M_2 / p_2) - \lambda_2 p_2 = 0\\
&p_2 y_2 \le M_2.
\end{aligned}
$$

[illegible] $v_c(y_2) = \alpha v_c(M_2 / p_2)$ [illegible]

$$
\begin{aligned}
&v_c(y_2) = 1/y_2 = \alpha v_c(M_2 / p_2) = \alpha p_2 / M_2\\
&\Rightarrow p_2 = M_2 / \alpha y_2 > M_2 / y_2
\end{aligned}
$$

[illegible]

[illegible] $p_2 = M_2 / y_2$ ■

[illegible]

$$
\begin{aligned}
&v_c(c_2) - \lambda_1 p_2 - \lambda_2 p_2 = 0\\
&\alpha v_c(nw_3 / p_2) / p_2 - \lambda_1 = 0\\
&p_2 c_2 + w_3 \le p_2 y_2 + nw_2\\
&p_2 c_2 \le m_2.
\end{aligned}
$$

[illegible] $p_2 = M_2 / y_2$ [illegible]

$$
\begin{aligned}
&c_2 = m_2 / p_2 = m_2 y_2 / M_2\\
&nw_3 = M_2 + \left(tb_2 - TB_2\right)(1 + r_1)
\end{aligned}
$$

[illegible]

$$
m_2 < nw_3 / \alpha \text{ with } nw_3 = M_2 + (tb_2 - TB_2)(1 + r_1).
$$

[illegible]

$$
\begin{aligned}
U(m_2, tb_2) &= \ln(c_2) + \alpha \ln(nw_3 / p_2) \\
&= \ln(m_2 f / M_2) + \alpha \ln(nw_3 f / M_2) \\
&= (1+\alpha)\ln(f) + \ln(m_2 / M_2) + \alpha \ln(nw_3 / M_2).
\end{aligned}
$$

[illegible]

$$
\begin{aligned}
c_2 &= nw_3 / \alpha p_2 = nw_3 y_2 / \alpha M_2 \\
nw_3 &= \frac{\alpha}{1+\alpha}\left[M_2 + m_2 + (tb_2 - TB_2)(1 + r_1)\right]
\end{aligned}
$$

[illegible]

$$
\begin{aligned}
U(m_2, tb_2) &= \ln(c_2) + \alpha \ln(nw_3 / p_2) \\
&= \ln(nw_3 f / \alpha M_2) + \alpha \ln(nw_3 f / M_2) \\
&= (1+\alpha)\ln(f) + (1+\alpha)\ln(nw_3 / M_2) - \ln(\alpha). \blacksquare
\end{aligned}
$$

[illegible]

$$
\begin{aligned}
& u_c(c_1, h) - \lambda_1 p_1 - \lambda_2 p_1 = 0 \\
& u_h(c_1, h) - \lambda_1 q = 0 \\
& \beta U_m - \lambda_1 = 0 \\
& \beta U_{tb} - \lambda_1 = 0 \\
& p_1 c_1 + qh + m_2 + tb_2 \le p_1 y_1 + qH + nw_1 \\
& p_1 c_1 \le m_1.
\end{aligned}
$$

[illegible]

$$
\begin{aligned}
U_m(m_2 = M_2, tb_2 = TB_2) &= 1 / M_2 \\
U_{tb}(m_2 = M_2, tb_2 = TB_2) &= \alpha(1 + r_1) / M_2.
\end{aligned}
$$

[illegible]

$$
\begin{aligned}
& \lambda_1 = \beta / M_2, 1 + r_1 = 1/\alpha, q = M_2 / \beta H \\
& 1/y_1 - \lambda_1 p_1 - \lambda_2 p_1 = 0 \\
& p_1 y_1 \le M_1.
\end{aligned}
$$

[illegible]

$$p_1 = M_1 / y_1$$
$$1/y_1 - \lambda_1 p_1 = \lambda_2 p_1 > 0 \Rightarrow M_2 > M_1\beta.$$

$$\lambda_2 = 0$$
$$p_1 = 1/y_1\lambda_1 = M_2/\beta y_1$$
$$p_1 y_1 = M_2/\beta < M_1 \Rightarrow M_2 < M_1\beta.$$

■

$$y_2 = \begin{cases} \dfrac{A[\beta NW + (\delta\alpha/(1-\alpha) - \beta)M_2]y_1}{M_2} & \text{if } M_2 < M_1\beta \\ \dfrac{A[\beta NW + (\delta\alpha/(1-\alpha) - \beta)M_2]y_1}{\beta M_1} & \text{if } M_2 > M_1\beta. \end{cases}$$

■

$$\pi = p_2/p_1 = \begin{cases} \dfrac{\beta y_1}{A\dfrac{[\beta NW + (\delta\alpha/(1-\alpha) - \beta)M_2]y_1}{M_2}} & \text{if } M_2 < M_1\beta \\ \dfrac{M_2 y_1}{M_1 A\dfrac{[\beta NW + (\delta\alpha/(1-\alpha) - \beta)M_2]y_1}{\beta M_1}} & \text{if } M_2 > M_1\beta \end{cases}$$

$$= \frac{\beta M_2}{A\left[\beta NW + (\delta\alpha/(1-\alpha) - \beta)M_2\right]}.$$

■

$$\frac{(1+\alpha)(\delta\alpha/(1-\alpha) - \beta)y_1}{M_1 L(M_2)}$$

$$\frac{\gamma}{NW - M_2}$$

$$\frac{(1+\alpha)(\delta\alpha/(1-\alpha) - \beta)y_1}{M_1 L(M_\psi)} = \frac{\gamma}{NW - M_\psi}$$

$M_2 > M_1\beta$ $M_\psi > M_\pi$ ■

$M_\psi > M_\pi$

$M_\psi > M_\pi > M_1\beta$

$M_2 > M_1\beta$

$M_\psi > M_1\beta > M_\pi$ … $M_2 < M_1\beta$ …

… $M_2 > M_1\beta$ …

$M_1\beta > M_\psi > M_\pi$ … $M_2 > M_1\beta$ …

… $M_2 < M_1\beta$ … ■

$M_\psi > M_\pi > M_1\beta$ … $M_1\beta > M_\psi > M_\pi$ …

$M_\psi > M_1\beta > M_\pi$ … $M_\psi > M_1\beta > M_\pi$ …

$$V(M_\pi \mid M_1) = \ln y_1 + \ln H + \beta(1+\alpha)\ln\frac{A\left[\beta NW + (\delta\alpha/(1-\alpha)-\beta)M_\pi\right]y_1}{M_\pi} - \gamma\ln\frac{NW}{NW - M_\pi},$$

$$V(M_\psi \mid M_1) = \ln y_1 + \ln H + \beta(1+\alpha)\ln\frac{A\left[\beta NW + (\delta\alpha/(1-\alpha)-\beta)M_\psi\right]y_1}{\beta M_1} - \gamma\ln\frac{NW}{NW - M_\psi}.$$

… $M_1 = M_\pi / \beta$ …

$M_1 = M_\psi / \beta$ … ■

$$X_{t+1} = E_B[u_c(y_{t+1}, H)/p_{t+1}]\Pr(B) + E_{NB}[u_c(y_{t+1}, H)c^{NB}_{w,t+1} + u_h(y_{t+1}, H)h^{NB}_{w,t+1}]\Pr(NB)$$

$$Z_{t+1} = E_B[u_h(y_{t+1}, H)(1+r_t)/q_{t+1}]\Pr(B) + E_{NB}[u_c(y_{t+1}, H)c^{NB}_{w,t+1}(1+r_t) + u_h(y_{t+1}, H)h^{NB}_{w,t+1}(1+r_t)]\Pr(NB)$$

$$X_{t+1} = E[u_c(y_{t+1}, H)/p_{t+1}]$$

$$Z_{t+1} = E[u_h(y_{t+1}, H)(1+r_t)/q_{t+1}]$$

$$p_t = M_t / y_t < NW / \underline{y}$$

$$p_t = M_t / y_t > \underline{M} / \bar{y}.$$

$$u(c_t,h_t)\le u(c_t^*,h_t^*)+\left(u_c(c_t^*,h_t^*)\quad u_h(c_t^*,h_t^*)\right)\begin{pmatrix}c_t-c_t^*\\ h_t-h_t^*\end{pmatrix}$$

$$=u(c_t^*,h_t^*)+\left(u_c(c_t^*,h_t^*)/p_t\quad u_h(c_t^*,h_t^*)/q_t\right)\begin{pmatrix}p_t(c_t-c_t^*)\\ q_t(h_t-h_t^*)\end{pmatrix}$$

$$\le u(c_t^*,h_t^*)+\left(u_h(c_t^*,h_t^*)/q_t\quad u_h(c_t^*,h_t^*)/q_t\right)\begin{pmatrix}p_t(c_t-c_t^*)\\ q_t(h_t-h_t^*)\end{pmatrix}$$

$$=u(c_t^*,h_t^*)+\beta X_{t+1}\left(p_t(c_t-c_t^*)+q_t(h_t-h_t^*)\right)$$

$$=u(c_t^*,h_t^*)+\beta X_{t+1}\left\{\left[(m_t-m_t^*)+(tb_t-tb_t^*)(1+r_{t-1})\right]-\left[(m_{t+1}-m_{t+1}^*)+(tb_{t+1}-tb_{t+1}^*)\right]\right\},$$

[illegible] $u_c(c_t^*,h_t^*)/p_t\ge u_h(c_t^*,h_t^*)/q_t$ [illegible] $c_t\le c_t^*$ [illegible]

[illegible]

$$\beta X_{t+1}\left[(m_t-m_t^*)+(tb_t-tb_t^*)(1+r_{t-1})\right]$$

$$=\left(u_h(c_t^*,h_t^*)/q_t\quad u_h(c_t^*,h_t^*)(1+r_{t-1})/q\right)\begin{pmatrix}m_t-m_t^*\\ tb_t-tb_t^*\end{pmatrix}$$

$$\le\left(X_t\quad Z_t\right)\begin{pmatrix}m_t-m_t^*\\ tb_t-tb_t^*\end{pmatrix},$$

[illegible] $u_c(c_t^*,h_t^*)/p_t\ge u_h(c_t^*,h_t^*)/q_t$ [illegible] $m_t\ge m_t^*$ [illegible]

[illegible]

$$u(c_t,h_t)\le u(c_t^*,h_t^*)+\left(X_t\quad Z_t\right)\begin{pmatrix}m_t-m_t^*\\ tb_t-tb_t^*\end{pmatrix}-\beta\left(X_{t+1}\quad Z_{t+1}\right)\begin{pmatrix}m_{t+1}-m_{t+1}^*\\ tb_{t+1}-tb_{t+1}^*\end{pmatrix}$$

[illegible]

$$\lim_{T\to\infty}E\left[\sum_{t=1}^{T}\beta^{t-1}[u(c_t,h_t)-u(c_t^*,h_t^*)]\right]$$

$$\le\lim_{T\to\infty}E\left[\sum_{t=1}^{T}\beta^{t-1}\left[\left(X_t\quad Z_t\right)\begin{pmatrix}m_t-m_t^*\\ tb_t-tb_t^*\end{pmatrix}-\beta\left(X_{t+1}\quad Z_{t+1}\right)\begin{pmatrix}m_{t+1}-m_{t+1}^*\\ tb_{t+1}-tb_{t+1}^*\end{pmatrix}\right]\right]$$

$$=\lim_{T\to\infty}E\left[-\beta^{T}\left(X_{T+1}\quad Z_{T+1}\right)\begin{pmatrix}m_{T+1}-m_{T+1}^*\\ tb_{T+1}-tb_{T+1}^*\end{pmatrix}\right]=0.$$

[illegible] $\{c_t^*,h_t^*,m_{t+1}^*,tb_{t+1}^*\}$ [illegible] ■

$[0, NW] \times [\underline{y}, \bar{y}]$

$$T_1(V)(M, y) = \max_{M'} \{u(y, H) - \psi(MBS / TB') + \beta E_{y'}[V(TB', y')]\}$$

$$\text{s.t.} \quad y' = f\left(\frac{TB' + \delta MBS}{p}, \varepsilon'\right)$$

$$T_q(p) = u_h(y, H) / \tilde{q}(TB, TB', y) = \beta E[u_c(y', H) / p(TB', TB'', y')]$$

$$T_p(p) = u_c(y, H) / \tilde{p}(TB, TB', y)$$

$$= \begin{cases} u_c(y/H)\, y/(NW - TB) & \text{if } u_c(y/H)\, y/(NW - TB) > u_h(y, H) / \tilde{q}(TB, TB', y) \\ T_q(p) = u_h(y, H) / \tilde{q}(TB, TB', y) & \text{otherwise} \end{cases}$$

$\tilde{p}$ $\tilde{q}$

$[0, NW] \times [\underline{y}, \bar{y}]$

$[0, NW]^2 \times [\underline{y}, \bar{y}]$

[illegible]

[illegible]

[illegible]

$$\underline{V}(M, y, M') \equiv \min_{(U', X', Z')} \left[u(c, h) - \psi(M, y) + \beta V'\right]$$

s.t. $\zeta = (M', V', X', Z')$ is consistent with respect to W at M

with c and h being the corresponding consumptions

[illegible]

$$u(c(y),h(y)) - \psi(M,y) + \beta V'(y) \geq \underline{V}(M,y)$$ [illegible]

[illegible]

[illegible]

[illegible]

$$\begin{aligned}\tilde{V}(M,\zeta) &\equiv E\left[u(c(y),h(y)) - \psi(M) + \beta V'(y)\right] \\ \tilde{X}(M,\zeta) &\equiv E\left[u_c(c(y),h(y)) / p(y)\right] \\ \tilde{Z}(M,\zeta) &\equiv E[u_h(c(y),h(y))(1+r_0)/q(y)]\end{aligned}$$

[illegible] $\Xi(M,\zeta) \equiv (\tilde{V}(M,\zeta), \tilde{X}(M,\zeta), \tilde{Z}(M,\zeta))$ [illegible]

$$B(W)(M) = \{\Xi(M,\zeta) \mid \zeta \text{ is admissible with respect to } W \text{ at } M\}$$

[illegible]

[illegible] $V(M) \in [(u(\underline{y},H) - \overline{\psi})/\beta, (u(\overline{y},H) - \underline{\psi})/\beta]$ [illegible]

[illegible]

[illegible]

[illegible]

[illegible] $\lim_{n\to\infty}\|g_n - g\|_p \to 0$ [illegible] $\lim_{n\to\infty}\|g_n - g\|_p \to 0$ [illegible]

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